Second-Order Analog Filters Having Truly Independent Tunability of Center Frequency and Bandwidth

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Introduction

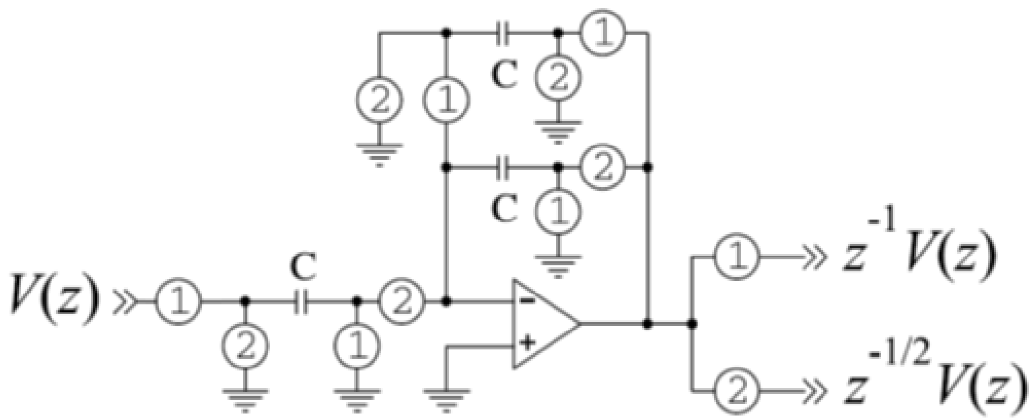
- In many integrated system applications, 1st and 2nd-order sections are required for higher order filter designs;
- Analog filter realization featuring independently tunable center frequency (ω_o) and 3-dB bandwidth (ω_b) was up to recently missing;
- Consequences:
  - Changes of ω_o also affects ω_b , and vice-versa;
  - The solution is the tuning of 2 or more components (usually C´s and R´s);
  - The cost is the increase of structure complexity, and consequently of silicon area;

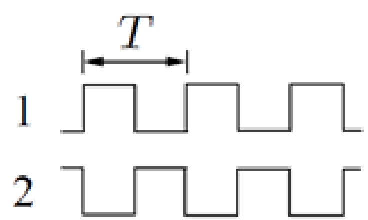
Introduction

- Independent tunability of ω_o and ω_b allows simple circuitry to program these two parameters, which leads to area reduction;
- In the analog domain, switched-capacitor (SC) techniques are particularly suitable:
  (i) their transfer functions are expressed in the z-domain;
  (ii) filter coefficients can be accurately implemented by capacitance ratios;
- Several strategies for 2nd-order SC filter realizations have been reported;
- Techniques to improve the accuracy of the filter coefficients are advanced.

Introduction

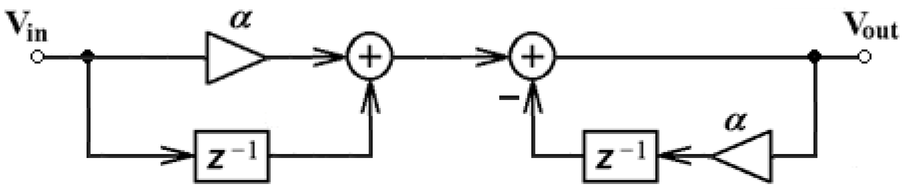
Realization of half and integer delays and multipliers





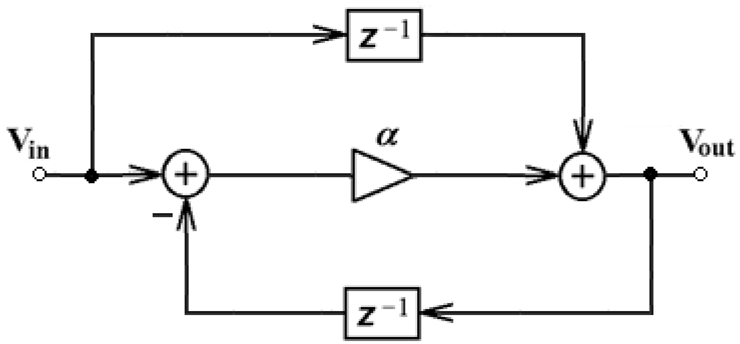
Discrete-Time Allpass Filters

1st-order allpass transfer function:



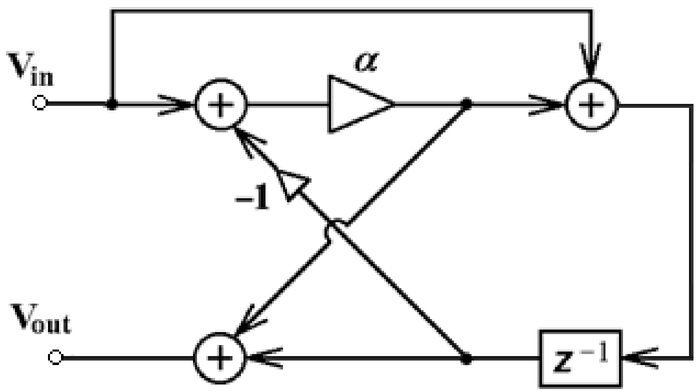
$$F(z) = \frac{V_{out}}{V_{in}}(z) = \frac{\alpha + z^{-1}}{1 + \alpha z^{-1}}$$

1st-order *structurally* allpass transfer function:



Discrete-Time Allpass Filters

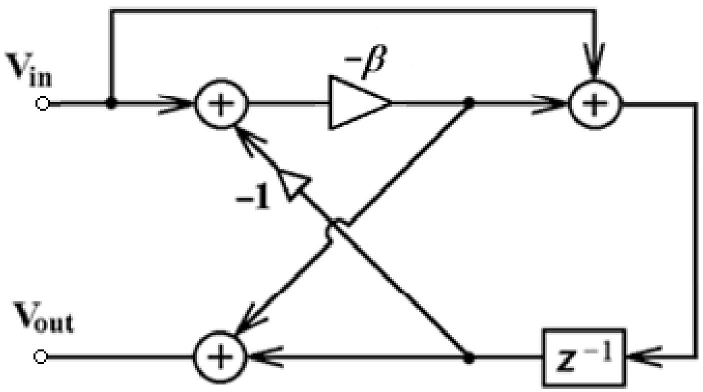
Alternative 1st-order structurally allpass transfer function using a lattice structure:



$$G(z) = \frac{V_{out}}{V_{in}}(z) = \frac{\alpha + z^{-1}}{1 + \alpha z^{-1}}$$

Discrete-Time Allpass Filters

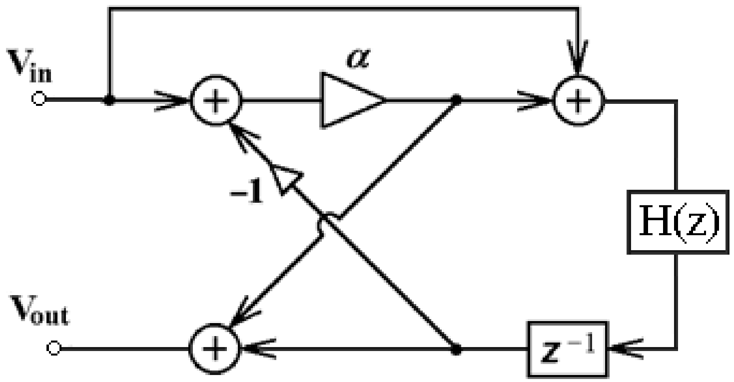
Alternative 1st-order structurally allpass transfer function using a lattice structure:



$$H(z) = \frac{V_{out}}{V_{in}}(z) = \frac{-\beta + z^{-1}}{1 - \beta z^{-1}}$$

Discrete-Time Allpass Filters

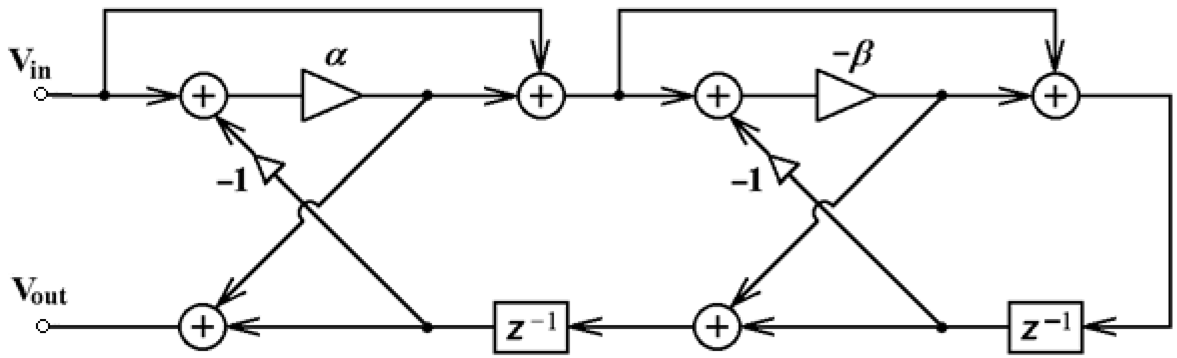
2^{nd}-order structurally allpass section implemented as a series connection of two 1^{st}-order allpass lattice networks:



$$A(z) = \frac{V_{out}}{V_{in}}(z) = \frac{H(z)z^{-1} + \alpha}{1 + \alpha H(z) z^{-1}}$$

Analog Discrete-Time Second-Order Allpass Filters

2^{nd}-order structurally allpass section implemented as a series connection of two 1^{st}-order allpass lattice networks:

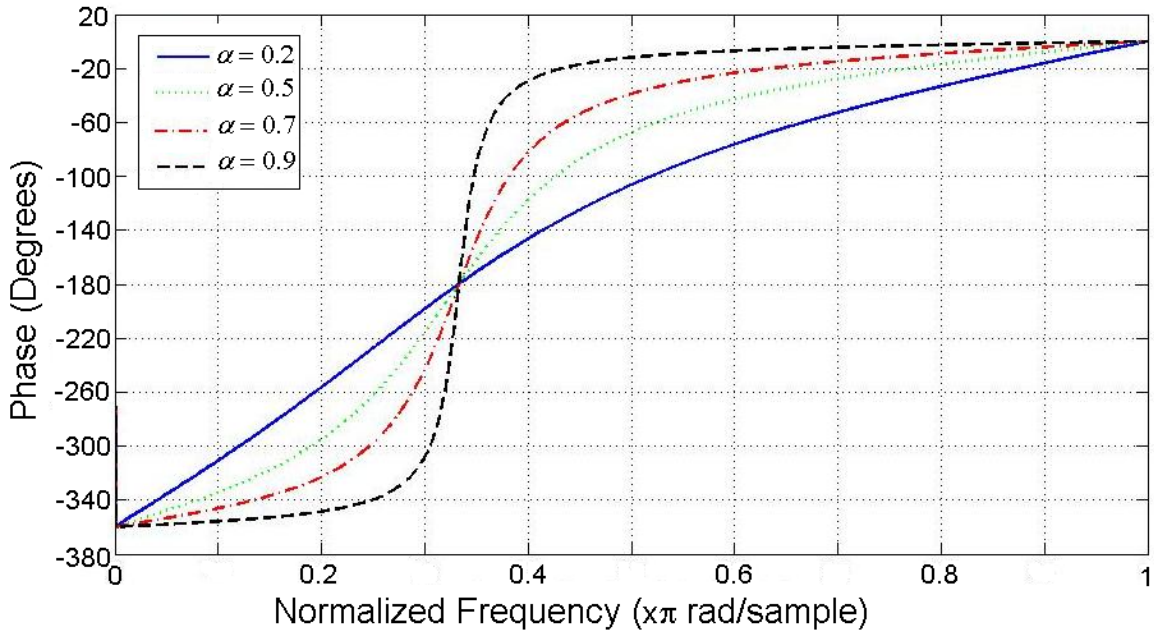


$$A(z) = \frac{V_{out}}{V_{in}}(z) = \frac{\alpha - \beta(1+\alpha)z^{-1} + z^{-2}}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}}$$

- α establishes the rate with which the fase of $A(e^{j\omega})$ crosses -180°, and hence determines the 3-dB bandwidth: $\omega_b = \cos^{-1}(2\alpha/(1+\alpha^2))$
- β determines the frequency at -180°: $\omega_0 = \cos^{-1}(\beta)$

Analog Discrete-Time Second-Order Allpass Filters

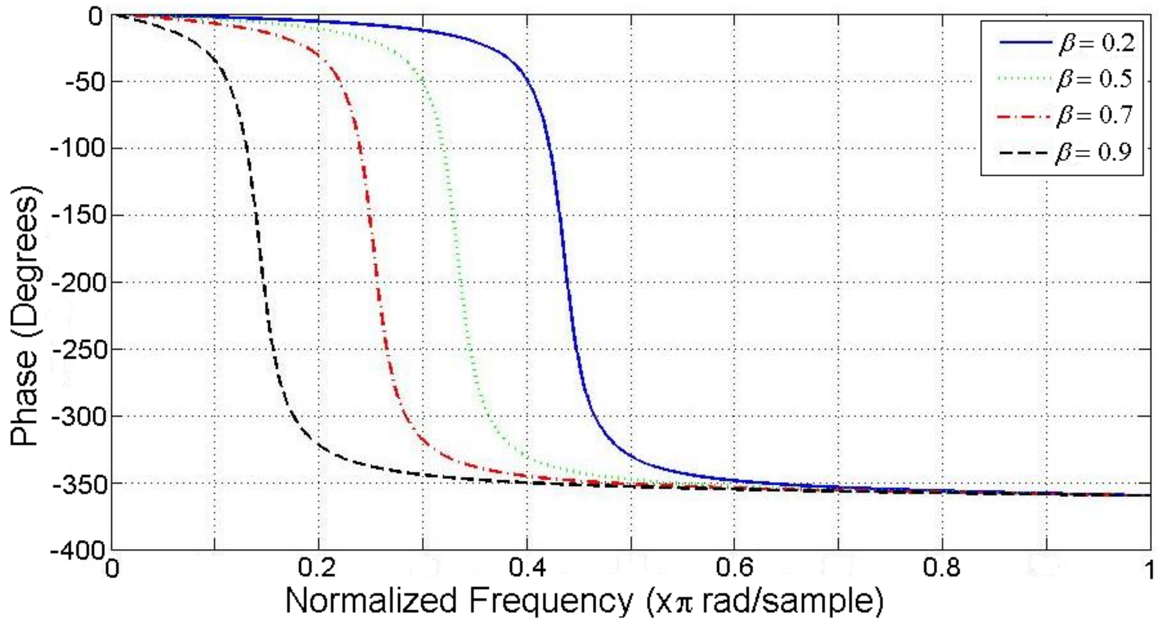
Central frequency fixed at $\omega_0 = 0.333\pi$ ($\beta = 0.5$).



- α establishes the rate with which the fase of $A(e^{j\omega})$ crosses -180º and hence determines the 3-dB bandwidth: $\omega_b = \cos^{-1}(2\alpha / (1+\alpha^2))$
- As α increases the filter bandwidth becomes narrower.

Analog Discrete-Time Second-Order Allpass Filters

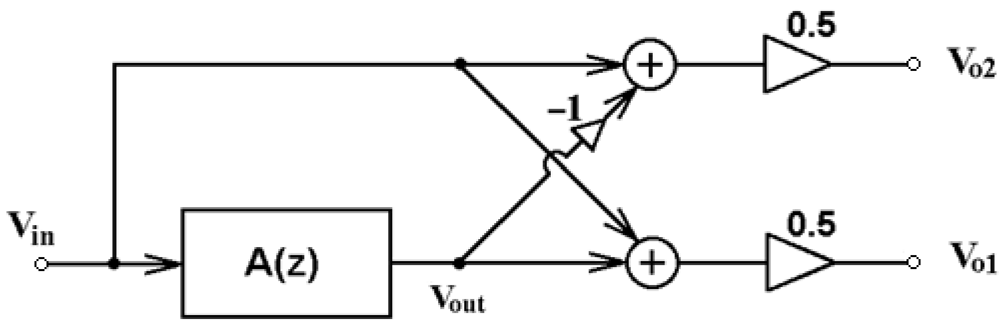
Fase rate fixed at $\omega_b = 0.205\pi$ ($\alpha = 0.5$).

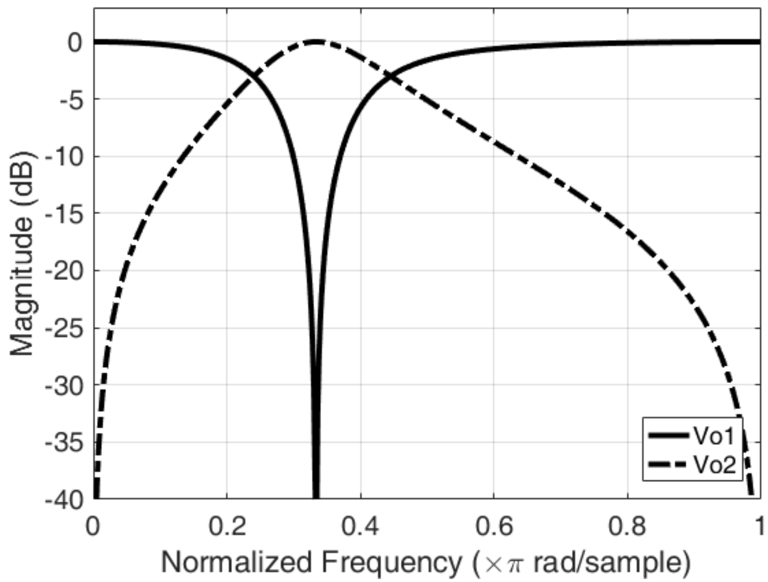


- β determines the center frequency, that is, the frequency at which the phase is -180º: $\omega_0 = \cos^{-1}(\beta)$

Analog Discrete-Time Second-Order Allpass Filters

By adding or subtracting the input and the output of the allpass section, either a bandstop or bandpass filter, respectively, is obtained:



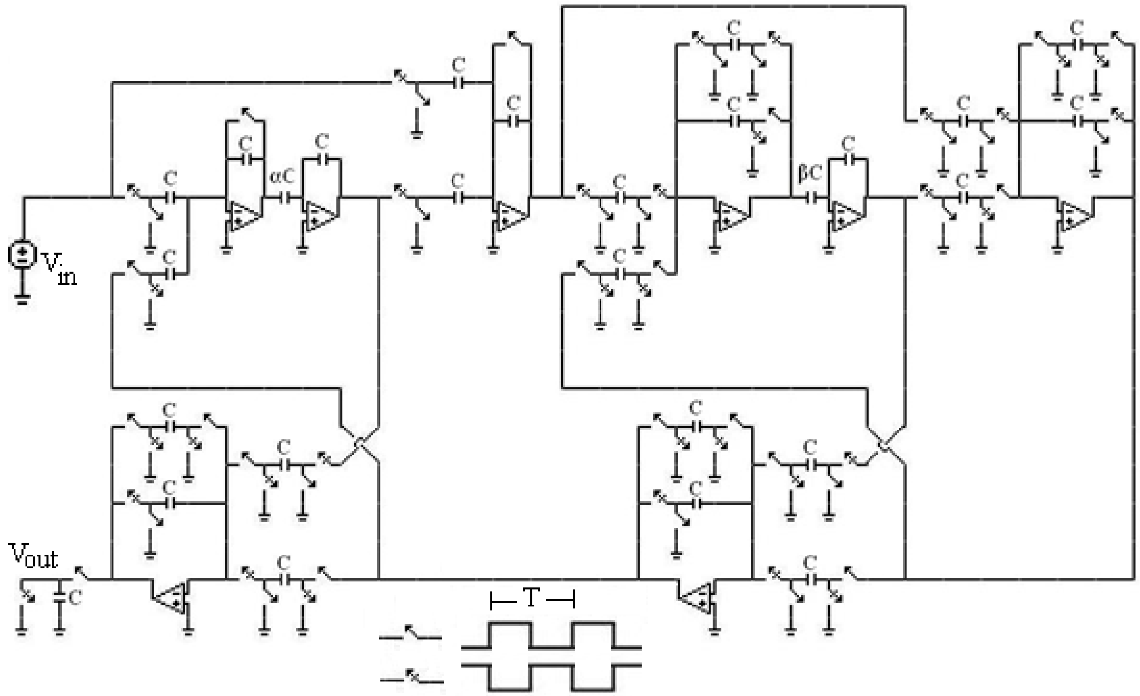


$\alpha = \beta = 0.5$

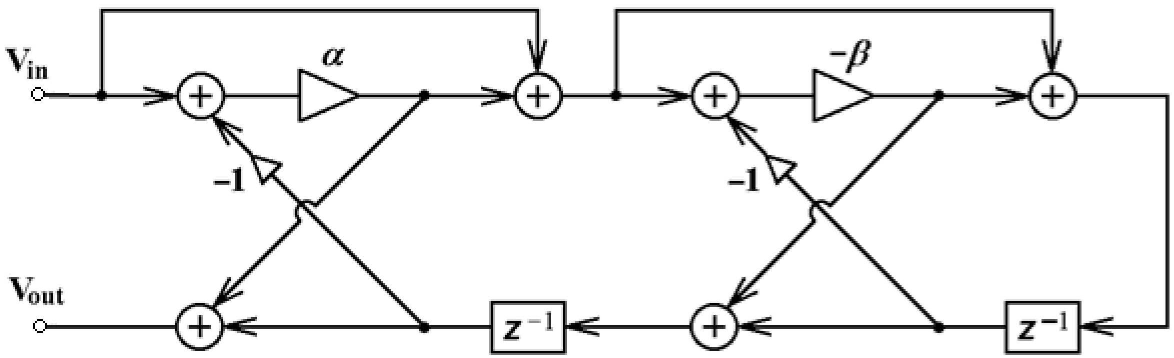
$\omega_0 = 0.333\pi$

$\omega_b = 0.205\pi$

SC Realization

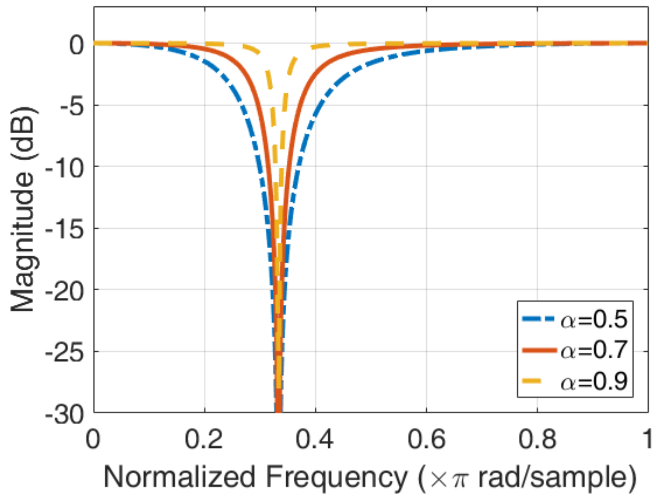


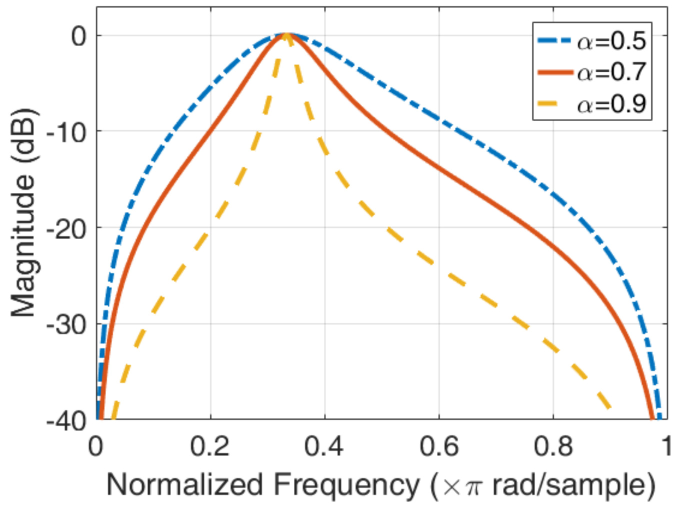
Structurally allpass lattice SC filter



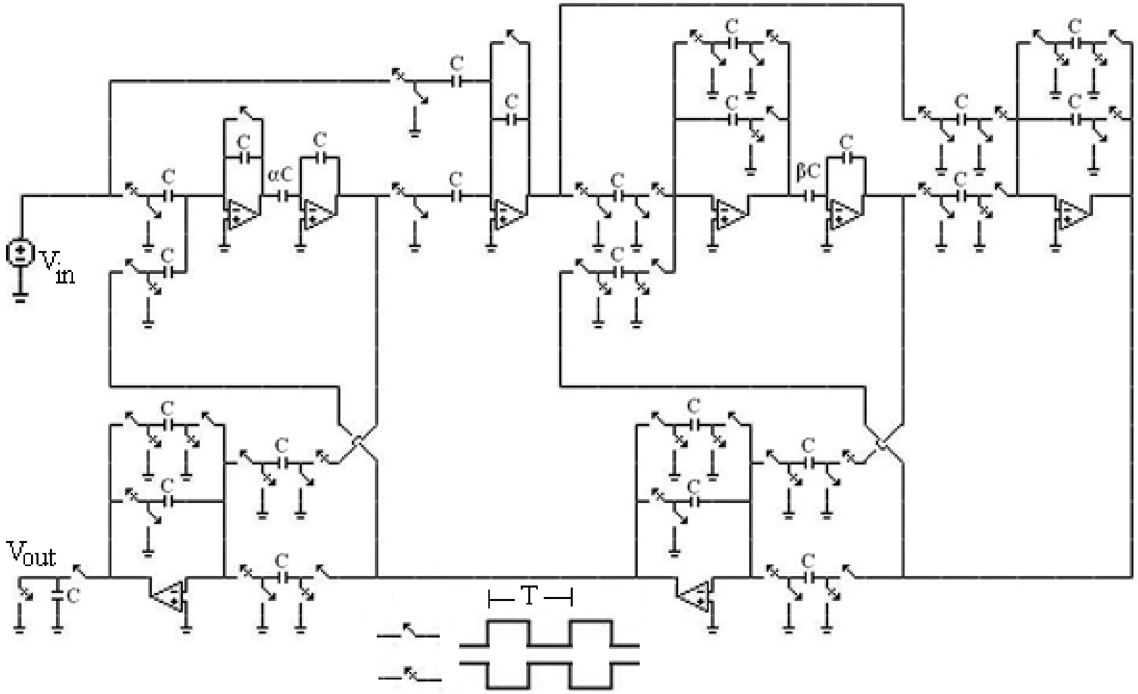
Circuit Simulations

Independent control of bandwidth (α) for a fixed central frequency ($\beta = 0.5$)

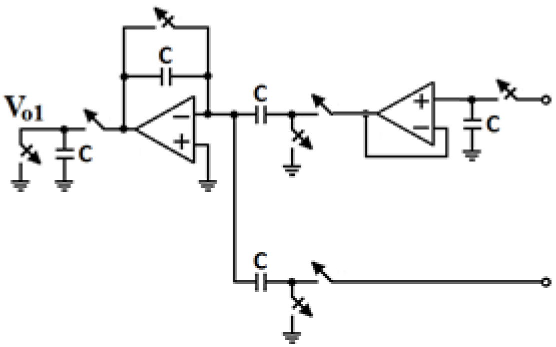




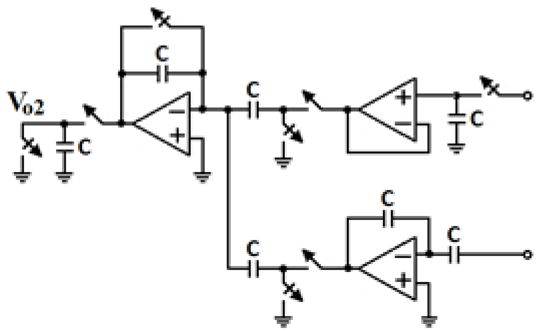
SC Realization



Structurally allpass lattice SC filter



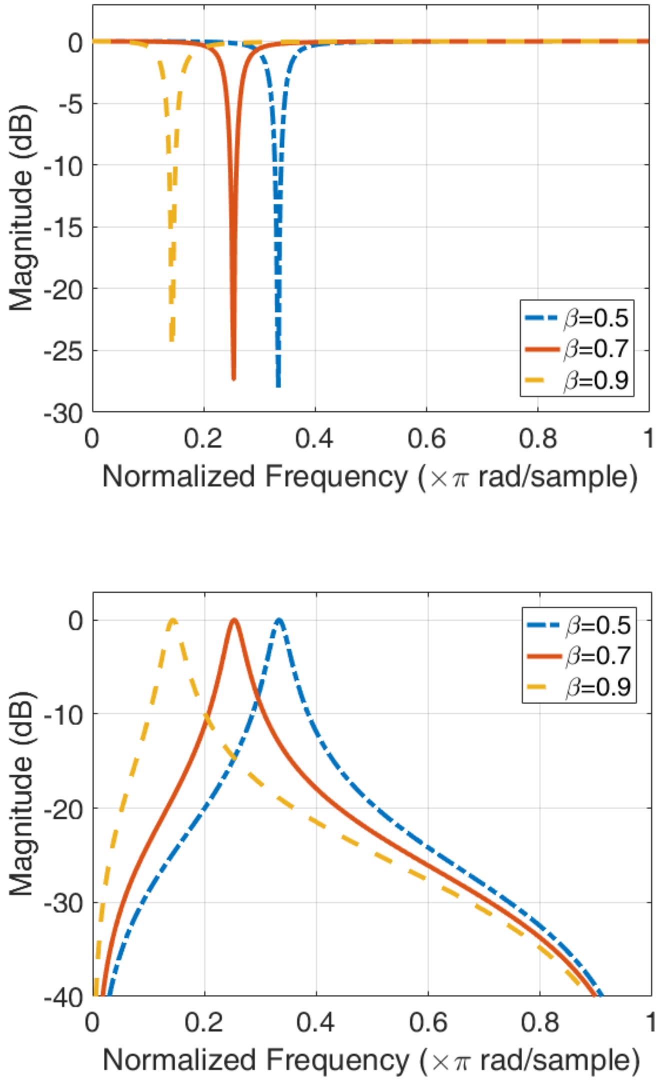
Adder



Subtractor

Circuit Simulations

Independent control of central frequency (β) for a fixed bandwidth ($\alpha = 0.9$)



Comparisons

Proposed			Fleischer & Laker											
Center Frequency Tuning														
α	β	**Total Cap.**	***A***	***B***	***C***	***D***	***E***	***G***	***H***	***I***	***J***	***K***	***L***	**Total Cap.**
0.95	0.2	155.75	1	1	1.56	1	0.05	2	2	1	1	0.025	0.256	435.64
0.95	0.5	62.90	1	1	0.975	1	0.05	2	2	1	1	0.025	0.256	412.24
0.95	0.7	45.21	1	1	0.585	1	0.05	2	2	1	1	0.025	0.256	396.64
0.95	0.9	35.39	1	1	0.195	1	0.05	2	2	1	1	0.025	0.256	381.04
Bandwidth Tuning														
0.2	0.5	61.40	1	1	0.6	1	0.8	2	2	1	1	0.40	0.265	43.19
0.5	0.5	62.00	1	1	0.75	1	0.5	2	2	1	1	0.25	0.265	43.02
0.7	0.5	62.40	1	1	0.85	1	0.3	2	2	1	1	0.15	0.265	70.73
0.9	0.5	62.80	1	1	0.95	1	0.1	2	2	1	1	0.05	0.265	207.12

Fleischer & Laker approach requires large capacitances for center frequency tuning, large capacitance spread (62.4) and simultaneous adjustments on 3 capacitances ***C***, ***E*** and ***K*** for bandwidth tuning.

Frequency Response Sentitivity to α and β

Relative error in the magnitude frequency response caused by simultaneous variations of α and β can be expressed as:

$$\frac{\Delta|H(\omega)|}{|H(\omega)|} = \frac{\Delta\alpha}{\alpha} S_{\alpha}^{|H(\omega)|} + \frac{\Delta\beta}{\beta} S_{\beta}^{|H(\omega)|}$$

where

$$S_{x}^{|H(\omega)|} = \frac{x}{|H(\omega)|} \frac{\partial|H(\omega)|}{\partial x}$$

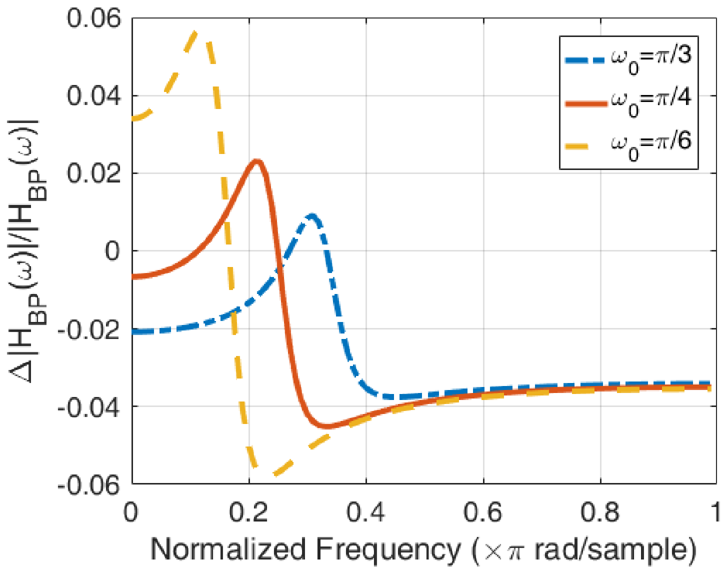
We assume the capacitance ratios have relative errors

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta\beta}{\beta} = \varepsilon = 1\%$$

Frequency Response Sentitivity to α and β

Bandpass Filter

$$\frac{\Delta|H_{BP}(\omega)|}{|H_{BP}(\omega)|}=\frac{\left(\frac{-2\alpha}{1-\alpha^2}(\cos\omega-\beta)+\beta\right)(\cos\omega-\beta)}{(\cos\omega-\beta)^2+\left(\frac{1-\alpha}{1+\alpha}\sin\omega\right)^2}\cdot\varepsilon$$

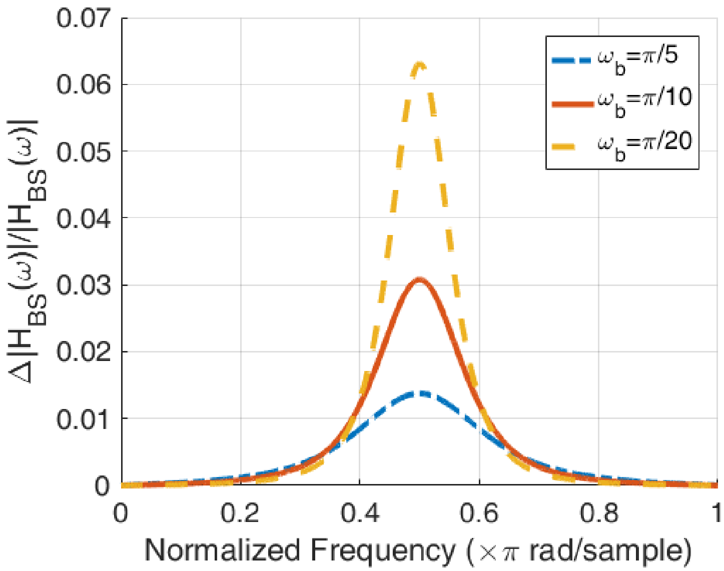


$\omega_b=\pi/10$

Frequency Response Sentitivity to α and β

Bandstop Filter

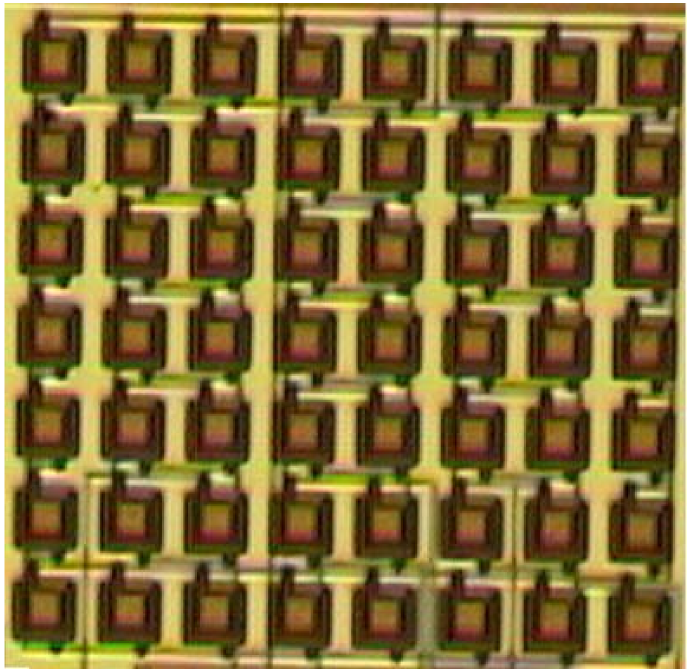
$$\frac{\Delta\left|H_{BS}(\omega)\right|}{\left|H_{BS}(\omega)\right|}=\frac{\left(\frac{1-\alpha}{1+\alpha}\sin\omega\right)\left(\frac{2\alpha}{(1+\alpha)^2}(\cos\omega-\beta)-\beta\frac{1-\alpha}{1+\alpha}\sin\omega\right)}{(\cos\omega-\beta)\left((\cos\omega-\beta)^2+\left(\frac{1-\alpha}{1+\alpha}\sin\omega\right)^2\right)}\cdot\varepsilon$$



$\omega_0=\pi/2$

Improving Filter Coefficient Accuracy

Techniques to improve capacitance ratio accuracy



Unit capacitors:

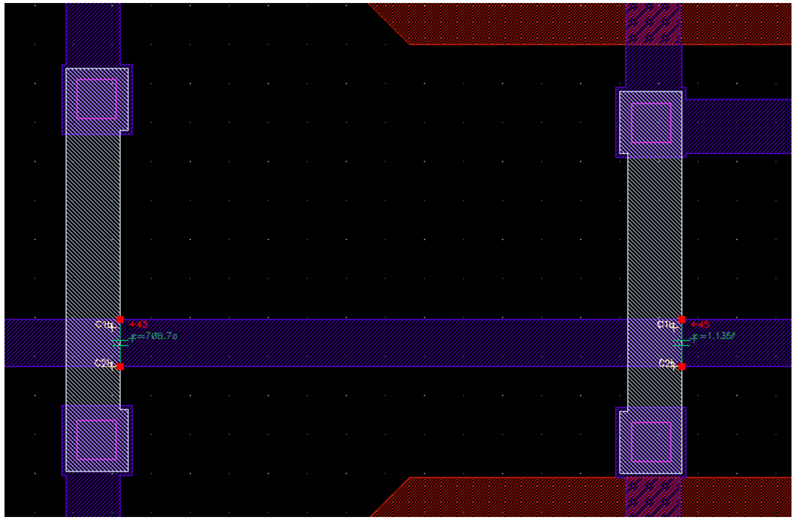
C = 100 fF

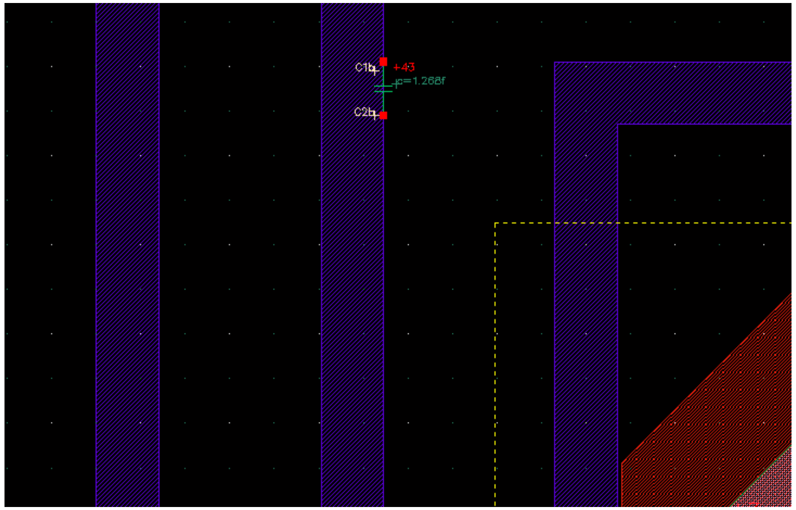
A = 5μm x 5μm

- Careful routing inside the capacitor array to avoid crossover and crosstalk, and therefore parasitic capacitances as well;
- Arrangement of unit capacitors in common centroid layout to reduce capacitance mismatch;

Improving Filter Coefficient Accuracy

crossover capacitances

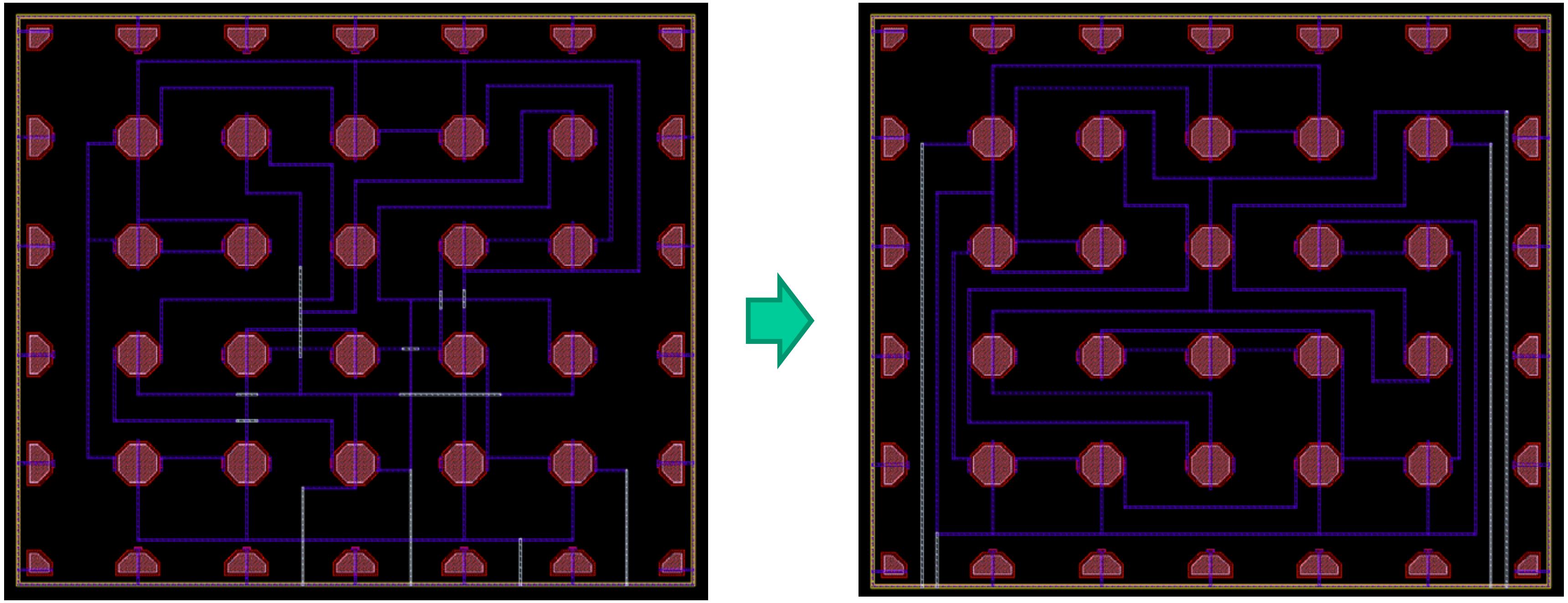




crosstalk capacitances

Improving Filter Coefficient Accuracy

Parasitic capacitances

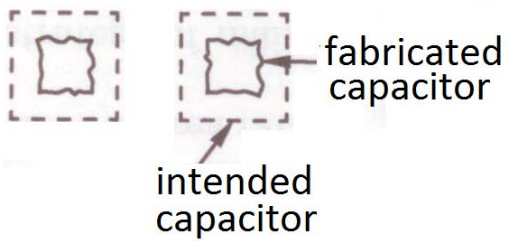


Crosstalk and crossover

Improving Filter Coefficient Accuracy

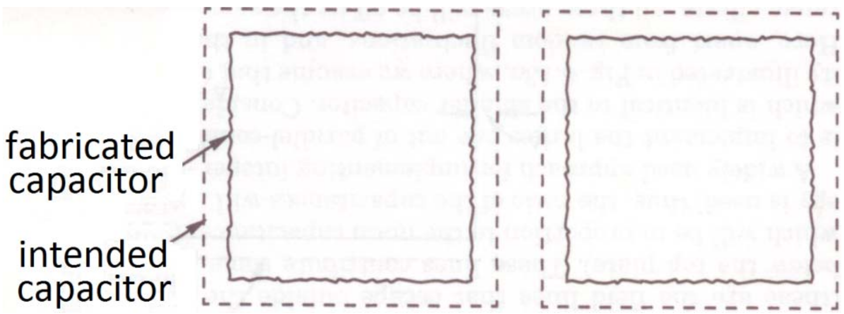
Small capacitors:

- The actual capacitance ratio can be significantly different from unity.



Large capacitors:

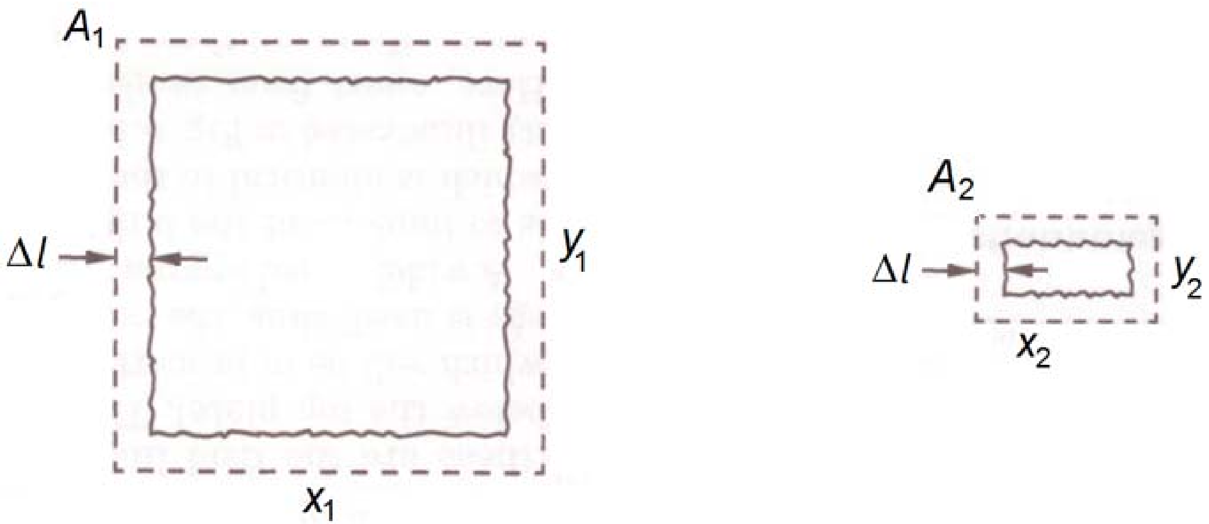
- Better ratio accuracy;
- However, if the plates are too large: (i) chip area may be excessive; (ii) opposite regions of the two capacitors may be affected differently by the fabrication process (e.g., slight difference in oxide thickness t_{ox});



Capacitance Ratio Error Sources

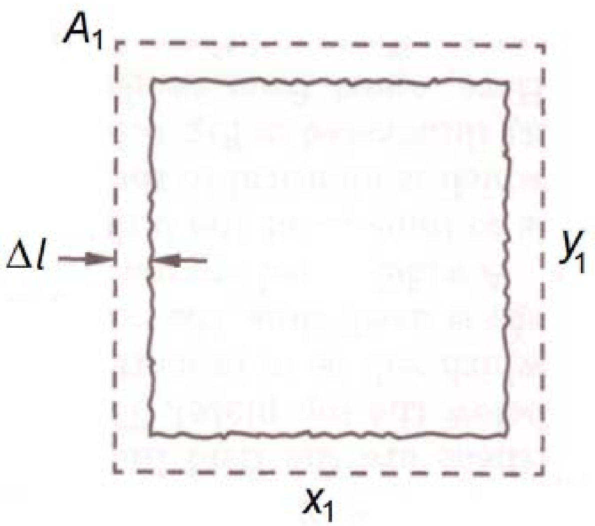
Systematic error caused by overetching

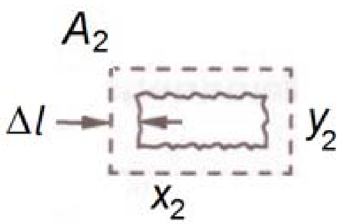
- It occurs when the upper layer of polysilicon or metal is being etched;
- The relative area errors of two capacitors will be the same if their nominal perimeter/area ratio is the same;
- Therefore the capacitance ratio will not be affected.



Capacitance Ratio Error Sources

Overetching effects





Let $A'_1 = (x_1 - 2\Delta l)(y_1 - 2\Delta l)$

$$\approx x_1 y_1 - 2(x_1 + y_1)\Delta l$$

$$= A_1 - P_1 \Delta l$$

The relative area error is

$$\frac{\Delta A_1}{A_1} = -\frac{P_1 \Delta l}{A_1}$$

The real capacitance ratio is

$$\frac{C'_1}{C'_2} = \frac{A'_1}{A'_2} = \frac{A_1\left(1 - \frac{P_1 \Delta l}{A_1}\right)}{A_2\left(1 - \frac{P_2 \Delta l}{A_2}\right)}$$

Therefore, if

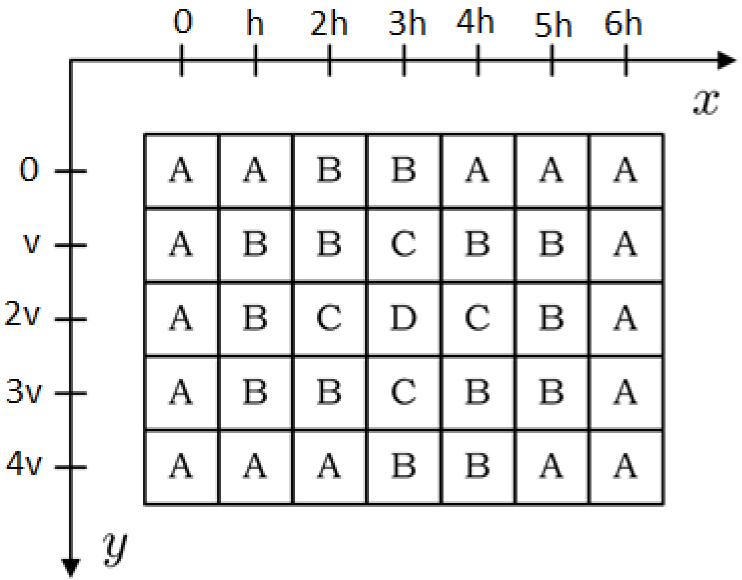
$$\frac{P_1}{A_1} = \frac{P_2}{A_2}$$

then

$$\frac{C'_1}{C'_2} = \frac{C_1}{C_2}$$

Improving Filter Coefficient Accuracy

Symmetrical layout with common centroid - evaluation of the average capacitance of each capacitor, assuming a linear model for t_{ox} variation:



$$\langle C_{xy}\rangle_A = \frac{1}{16}(16C_u + 48C_h + 32C_v) = C_u + 3C_h + 2C_v$$

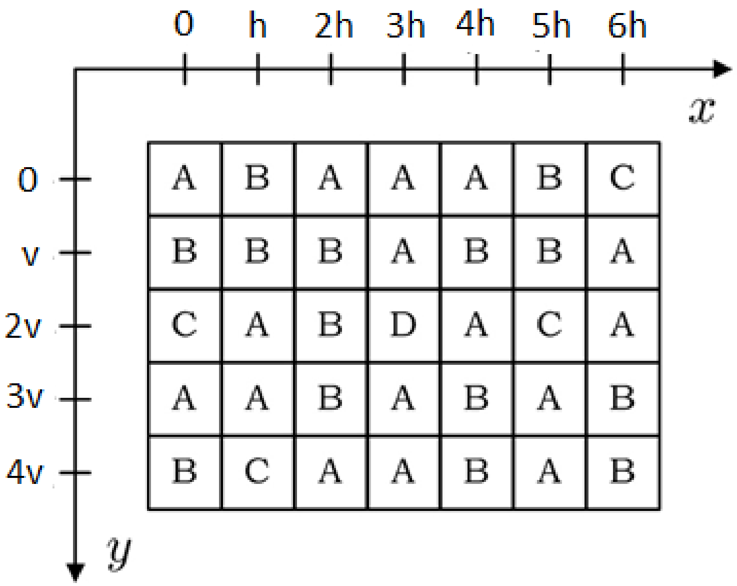
$$\langle C_{xy}\rangle_B = \frac{1}{14}(14C_u + 42C_h + 28C_v) = C_u + 3C_h + 2C_v$$

$$\langle C_{xy}\rangle_C = \frac{1}{4}(4C_u + 12C_h + 8C_v) = C_u + 3C_h + 2C_v$$

$$\langle C_{xy}\rangle_D = C_u + 3C_h + 2C_v$$

Capacitor Layout

Asymmetrical layout with common centroid:



$$\langle C_{xy} \rangle_A = \frac{1}{16}(16C_u + 48C_h + 32C_v) = C_u + 3C_h + 2C_v$$

$$\langle C_{xy} \rangle_B = \frac{1}{14}(14C_u + 42C_h + 28C_v) = C_u + 3C_h + 2C_v$$

$$\langle C_{xy} \rangle_C = \frac{1}{4}(4C_u + 12C_h + 8C_v) = C_u + 3C_h + 2C_v$$

$$\langle C_{xy} \rangle_D = C_u + 3C_h + 2C_v$$

Capacitor Layout

Arrangement of unit capacitors to reduce capacitance mismatch

- Common centroid arrangement is not a simple task when the number of capacitance ratios (i.e. coefficients) is large;
- Common centroid layout is not always possible;
- When possible, there are several alternatives;
- A choice can thus be made to avoid or reduce crossover and crosstalk parasitic capacitances.
- Common centroid layout tends to increase the spatial correlation coefficients of the capacitors;
- Find the optimal arrangement that minimizes the common centroid error and maximizes the spatial correlation.

Conclusions

- A new approach for the design of 2nd-order bandpass and bandstop SC filters was presented;
- The center frequency and 3-dB bandwidth are tuned independently by only two capacitance ratios, thereby reducing capacitance spread, circuit area and power consumption;
- The circuit core is a structurally allpass SC filter, and therefore regardless of coefficient error realizations the allpass transfer function property is preserved;
- Frequency response sensitivity to coefficient errors is thus reduced;
- A sensitivity analysis was conducted to verify the low frequency response variations with respect to capacitance ratio errors, for both bandpass and bandstop filters;
- Arrangements of unit capacitors to reduce capacitance mismatch were shown.