Effects of Coefficient Inaccuracy in Switched-Capacitor Transversal Filters

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***Abstract*—Coefficient inaccuracy effects in the frequency response of FIR switched-capacitor filters implemented in direct form are investigated. No assumption is made with respect to the filter coefficients, so that the results of the analysis are valid for both linear and nonlinear phase FIR filters. Assuming that the errors in the capacitor ratios realizing the tap coefficients are uncorrelated and have identical Gaussian distributions, an exact probability distribution function is derived for the error in the frequency response. It is shown that this distribution can be characterized by a Rayleigh distribution, which is then used to derive an upper bound for the expected stopband attenuation. Extensive simulation results are shown, as well, to give support to the analysis.**

I. Introduction

Analog transversal filters have been implemented in a number of technologies, including charge-coupled devices and charge-transfer devices. These techniques allow for the implementation of discrete-time transversal filters without the need for analog-to-digital converters. The availability of high-quality MOS capacitors and switches has made switched-capacitor (SC) networks an increasingly attractive alternative, which is reflected in many applications reported in the related literature [1]–[6]. Also, a high-speed GaAs transversal filter has been proposed in [7], as a result of the growing interest in the use of this technology for implementing SC integrated filters. The structure of a transversal filter uses two basic building blocks: the unit-delay and the linear combiner, comprising the filter coefficients and the summer. In order to minimize or eliminate errors such as offset voltage, stray capacitances, gain variations, and clock feedthrough, several circuit techniques have been proposed for the realization of high-quality SC unit-delays [8]–[12], which are cascaded to implement the analog delay line, and SC adders [10], [13]–[15], which implement the linear combiner. In this paper we assume that the errors mentioned above have negligible effects, so that the filter transfer function can be written as $\sum_{k=0}^{N-1} h_k z^{-k}$, where N is the filter length and h_k is the kth filter coefficient. Since these coefficients are implemented by capacitor ratios, their limited accuracy may be the most important cause of deviation from the desired frequency response of SC transversal filters.[1] This is in contrast with the digital case where the filter-coefficients can be implemented with high precision by increasing the wordlength.

The accuracy of the capacitor ratios is affected by several factors such as voltage and temperature dependencies, parasitic capacitances, and errors that are inherent to the fabrication process. Errors due to voltage and temperature variations are in general negligible and easily avoided with a careful layout arrangement. Parasitic capacitances can be efficiently eliminated by properly designing the circuit so that all the capacitors connected to a virtual ground are switched to low impedance nodes having the same voltage as that of the virtual ground. The last type of error depends strongly on the process and the equipment used. For a typical MOS process, the global edge and oxide variations represent the crucial limitations on the achievable matching accuracy [16], [17]. It has been shown [18] that the capacitor ratio errors may reasonably be modeled as uncorrelated and normally (Gaussian) distributed random variables, having zero mean and standard deviation that typically lies in the range of 0.001 to 0.05 [19].

Although the capacitor ratio errors affect the frequency response of a transversal filter in all frequencies, it is in the stopband that their effect is more apparent. The reason for this is that in this region the desired magnitude is very small as a result of a very delicate combination of all the tap signals [22]. This is illustrated in Fig. 1 where we show the frequency responses of three FIR filters (broken lines) whose coefficients have additive Gaussian errors with zero mean and standard deviation $\sigma_\epsilon = 0.001$. The ideal frequency response obtained with "infinite" precision coefficients is also shown (solid line). Observe that although the desired stopband attenuation is 107 dB,

Manuscript received September 10, 1990; revised January 16, 1990. This work was supported in part by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, and in part by a University of California Micro Grant, with matching support from the Rockwell International Corp.

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IEEE Log Number 9101483.

[1] This assumption may not be valid when N is very large. In such cases the accumulation of errors resulting from the signal being successively transferred from one stage to another of the delay line may not be tolerable.

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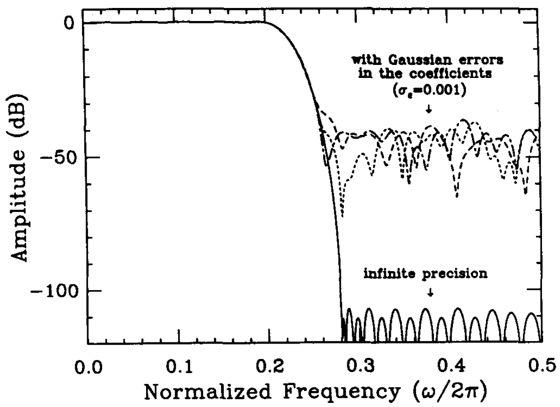

Fig. 1. FIR frequency responses; $N = 50$, $\Delta f = 0.08$, and $\delta_1 = 0.0122$.

an actual SC implementation with the state-of-the-art technology can only provide around 45 dB of attenuation.

The issue of the coefficient inaccuracy effects in transversal filters has been studied by several authors in the past for both digital and analog transversal structures [20]–[27]. In [26], in particular, it is shown that for large N, the maximum error in the frequency response of the magnitude of a linear phase transversal filter is given by $\sigma\sqrt{N \ln N}$, where σ is the standard deviation of the coefficient errors. This result is based on the assumption that the coefficient errors are independent identically distributed random variables with a zero mean and finite sixth-order moment. This result has been found to be moderately accurate in one study [27] of a low-pass filter with $N = 33$.

In this paper we investigate coefficient inaccuracy effects in the frequency response of SC transversal filters. We assume that the coefficient errors are independent and identically distributed Gaussian random variables, and derive an exact probability distribution function (pdf) for the error in the frequency domain. Since no particular assumption is made regarding the filter coefficients, the analysis is valid for both linear and nonlinear phase FIR filters.

II. Statistical Analysis

We begin our approach by writing the actual filter transfer function, on the unit circle, as

$$\hat{H}(\omega) = \sum_{k=0}^{N-1} (h_k + \epsilon_k) e^{j\omega k} = H(\omega) + \sum_{k=0}^{N-1} \epsilon_k e^{j\omega k} \tag{1}$$

where ϵ_k is a random error and represents the capacitance ratio fluctuation around the nominal value of the kth coefficient h_k. From the last equality we define the *error transfer function* as

$$\Delta H(\omega) = \hat{H}(\omega) - H(\omega) = \sum_{k=0}^{N-1} \epsilon_k e^{j\omega k} \tag{2}$$

which can be rewritten as

$$\Delta H(\omega) = A(\omega) + jB(\omega) \tag{3}$$

where

$$A(\omega) = \sum_{k=0}^{N-1} \epsilon_k \cos(\omega k), \qquad B(\omega) = \sum_{k=0}^{N-1} \epsilon_k \sin(\omega k). \tag{4}$$

Assuming that the ratio errors ϵ_k are uncorrelated Gaussian random variables and identically distributed with zero mean and standard deviation σ_ϵ, we have

$$\sigma_a^2(\omega) = E\{A^2(\omega)\} = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} E\{\epsilon_k \epsilon_l\} \cos(\omega k)\cos(\omega l) = \frac{\sigma_\epsilon^2}{4}\left(2N + 1 + \frac{\sin((2N-1)\omega)}{\sin\omega}\right) \tag{5}$$

$$\sigma_b^2(\omega) = E\{B^2(\omega)\} = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} E\{\epsilon_k \epsilon_l\} \sin(\omega k)\sin(\omega l) = \frac{\sigma_\epsilon^2}{4}\left(2N - 1 - \frac{\sin((2N-1)\omega)}{\sin\omega}\right) \tag{6}$$

and

$$\gamma(\omega) = E\{A(\omega)B(\omega)\} = \sum_{k=0}^{N-1}\sum_{l=0}^{N-1} E\{\epsilon_k \epsilon_l\} \cos(\omega k)\sin(\omega l) = \frac{\sigma_\epsilon^2}{4}\,\frac{\cos\omega + \cos((2N-1)\omega)}{\sin\omega} \tag{7}$$

where $E\{\cdot\}$ denotes expectation. For a given frequency ω, both $A(\omega)$ and $B(\omega)$ are zero mean Gaussian random variables, and therefore, they have a well-known joint pdf, which is given by

$$f_X(x) = \frac{1}{2\pi |C(\omega)|^{1/2}} \exp\left[-\frac{1}{2} x^T(\omega) C^{-1}(\omega) x(\omega)\right] \tag{8}$$

where

$$x^T(\omega) = [A(\omega), B(\omega)] \quad \text{and} \quad C(\omega) = E\{x(\omega)x^T(\omega)\} \tag{9}$$

and $|C(\omega)|$ is the determinant of the matrix $C(\omega)$. Notice that the above pdf is, as expected, frequency dependent, which means that the error introduced in the filter transfer function by mismatches in the capacitor ratios varies with frequency.

In much of the following work it is convenient to introduce another pair of random variables $R(\omega)$ and $\Theta(\omega)$, which are the magnitude and phase of the error

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transfer function, respectively, at the frequency ω, i.e.,

$$R(\omega)=\sqrt{A^2(\omega)+B^2(\omega)}\,, \qquad \Theta(\omega)=\arctan\left(\frac{B(\omega)}{A(\omega)}\right). \tag{10}$$

Now defining,

$$\kappa(\omega)=\frac{\cos((N-1)\omega)}{\sin\omega} \quad \text{and} \quad \nu^2(\omega)=|N^2-\kappa^2(\omega)| \tag{11}$$

we have the following identities from (5), (6), and (7):

$$\sigma_a^2(\omega)+\sigma_b^2(\omega)=N\sigma_\epsilon^2 \tag{12}$$

$$4|\sigma_a^2(\omega)\sigma_b^2(\omega)-\gamma^2(\omega)|=\sigma_\epsilon^4\nu^2(\omega) \tag{13}$$

$$\begin{aligned}&\left(\sigma_a^2(\omega)-\sigma_b^2(\omega)\right)\cos 2\theta+2\gamma(\omega)\sin 2\theta\\ &\quad=\sigma_\epsilon^2\kappa(\omega)\sin(2\theta+N\omega)\end{aligned} \tag{14}$$

where θ is an arbitrary angle. Using these identities we can write the joint pdf of $R(\omega)$ and $\Theta(\omega)$ as

$$\begin{aligned}f_{R\Theta}(r,\theta)&=\frac{r}{\pi\sigma_\epsilon^2\nu(\omega)}\exp\left[-\frac{Nr^2}{\sigma_\epsilon^2\nu^2(\omega)}\right]\\ &\quad\cdot\exp\left[\frac{r^2\kappa(\omega)\sin(2\theta+N\omega)}{\sigma_\epsilon^2\nu^2(\omega)}\right].\end{aligned} \tag{15}$$

Notice that, for simplicity of notation, we omitted the frequency dependence in r and θ, as we do in the following equations. Now the pdf of the magnitude of the error transfer function can be obtained as follows:

$$\begin{aligned}f_R(r)&=\int_0^{2\pi}f_{R\Theta}(r,\theta)\,d\theta\\ &=\frac{2r}{\sigma_\epsilon^2\nu(\omega)}\exp\left[-\frac{Nr^2}{\sigma_\epsilon^2\nu^2(\omega)}\right]I_0\left(\frac{r^2\kappa(\omega)}{\sigma_\epsilon^2\nu^2(\omega)}\right)\end{aligned} \tag{16}$$

where $I_0(\cdot)$ is the modified Bessel function of zero order.[2] Given the standard deviation σ_ϵ of the capacitance ratios used to implement an SC transversal filter of length N, then the probability that the error transfer function has, at the frequency ω, a magnitude $r_1 \leqslant |\Delta H(\omega)| \leqslant r_2$, is given by the area under $f_R(\cdot)$ in the range from r_1 to r_2.

We now turn our attention to the argument of I_0 in (16). First from (11) we observe that for a given filter of length N there exists a set $\Omega_N=(\omega_1,\omega_2)$ such that[3]

$$\kappa^2(\omega)\ll N^2 \quad \text{and} \quad \nu(\omega)^2\approx N^2, \qquad \forall\omega\in\Omega_N. \tag{17}$$

As a consequence,

$$I_0\left(\frac{r^2\kappa(\omega)}{\sigma_\epsilon^2\nu^2(\omega)}\right)\approx 1, \qquad \forall\omega\in\Omega_N. \tag{18}$$

[2] $I_0(x)=(1/2\pi)\int_0^{2\pi}\exp[x\cos\theta]\,d\theta$.

[3] In the limit, $\Omega_N \to \Omega_\infty=(0,0.5)$ as $N\to\infty$.

Substituting (17) and (18) in (16), we conclude that

$$f_R(r)\approx\frac{2r}{N\sigma_\epsilon^2}\exp\left[-\frac{r^2}{N\sigma_\epsilon^2}\right], \qquad \forall\omega\in\Omega_N \tag{19}$$

which is a Rayleigh distribution with mean and standard deviation given, respectively, by

$$\mu_r=\frac{\sqrt{\pi N}}{2}\sigma_\epsilon \quad \text{and} \quad \sigma_r=\frac{\sqrt{(4-\pi)N}}{2}\sigma_\epsilon. \tag{20}$$

Equation (19) shows that $|\Delta H(\omega)|$ has a distribution that is approximately independent of frequency for all frequencies in Ω_N.[4] In fact, as depicted in Fig. 2 for $N=8$, the pdf given by (16) is approximately frequency independent for all frequencies in the set $\Omega_8=(0.05,\ 0.45)$. In Fig. 3, the exact pdf in (16) and the Rayleigh pdf in (19) are shown for the frequencies $\omega=0.016$ and $\omega=0.3$. Observe that for frequencies outside Ω_8, the pdf tilts to the left, indicating a decrease in the mean value. This has been verified by computer simulations whose results we present later.

From the well-known inequality for complex numbers we can write

$$|\hat{H}(\omega)|\leqslant|H(\omega)|+|\Delta H(\omega)| \tag{21}$$

so that taking the expected value from both sides we obtain

$$E\{|\hat{H}(\omega)|\}\leqslant E\{|H(\omega)|\}+E\{|\Delta H(\omega)|\}. \tag{22}$$

Assuming that (19) is valid for all frequencies, i.e., $\Omega_N=\Omega_\infty$ (see footnote 2), and substituting (20) in (22) we get

$$E\{|\hat{H}(\omega)|\}\leqslant|H(\omega)|+\frac{\sqrt{\pi N}}{2}\sigma_\epsilon. \tag{23}$$

This equation shows how the expected value of the transfer function magnitude of the transversal filter is influenced by the error in its coefficients. In the passband the nominal magnitude of the frequency response is approximately *one*, and since σ_ϵ is typically small (~ 0.001) the filter length N has to be large before any influence is noticed. In fact, for $|H(\omega)|=1$ and $\sigma_\epsilon=0.001$ the two terms on the right-hand side of (23) have approximately the same value when $N\sim 10^6$. Based on this observation we neglect coefficient inaccuracy effects in the passband and focus our discussion on the stopband range of frequencies, where (23) can be rewritten as

$$E\{|\hat{H}(\omega)|\}\leqslant\delta_2+\frac{\sqrt{\pi N}}{2}\sigma_\epsilon. \tag{24}$$

An ideal filter would have $\delta_1=\delta_2=\Delta f=0$. In practice, however, one may have the passband ripple δ_1 and the transition band Δf fixed for a particular application and may have to choose the filter length N in order to achieve a desired stopband attenuation. If $\sigma_\epsilon=0$, meaning that there is no coefficient error, then $|\hat{H}(\omega)|=\delta_2$, and the stopband amplitude can be made as small as desired by

[4] Also the phase has an approximately frequency independent distribution, more specifically a *uniform* distribution with pdf $f_\Theta(\theta)=1/2\pi$, $\theta\in[0,2\pi]$.

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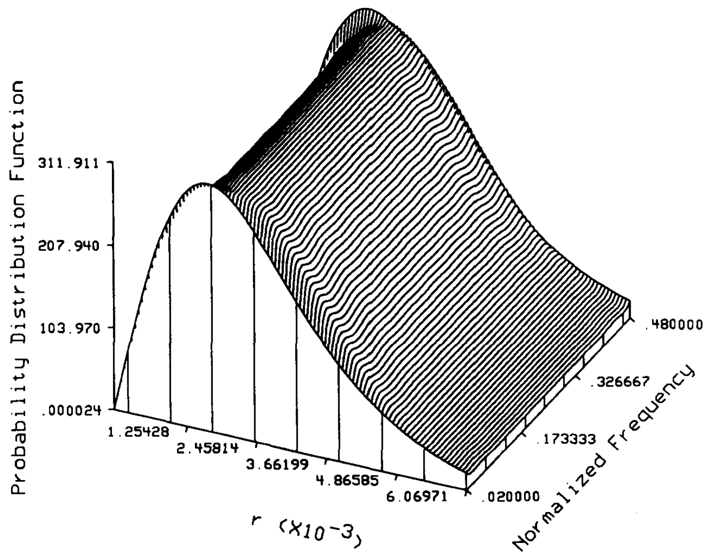

Fig. 2. Variation of the pdf of $|\Delta H(\omega)|$ (see (16)) with frequency; $N = 8$, $\sigma_\epsilon = 0.001$.

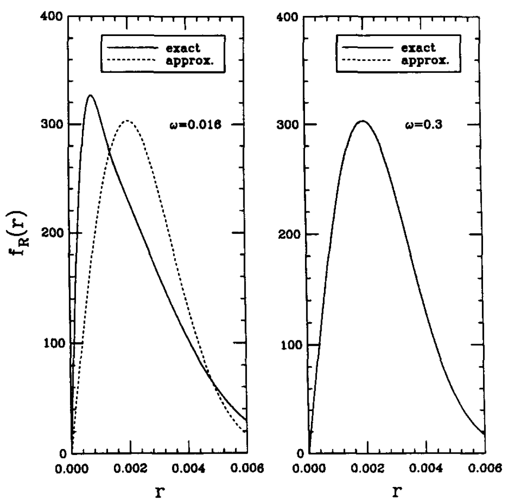

Fig. 3. Probability distributions of $|\Delta H(\omega)|$ for two particular frequencies.

increasing N. In the presence of coefficient errors, on the other hand, increasing the filter length does not lead to a decrease of the stopband ripple as an increase of degradation in the stopband frequency response also takes place, as indicated by (24). In other words, for a given coefficient accuracy, the larger the number of tap coefficients the lower the probability of obtaining the desired attenuation, since all the tap signals have to be precisely combined in order to produce a very small signal as an output. Even for moderate values of N the actual magnitude $\hat{\delta}_2$ may be far away from the nominal value δ_2, specially when this is required to be very small. In this case the actual magnitude is expected to be limited by the error term, i.e.,

$$E\{|\hat{H}(\omega)|\} \approx \frac{\sqrt{\pi N}}{2}\sigma_\epsilon, \quad \text{if} \quad \delta_2 \ll \frac{\sqrt{\pi N}}{2}\sigma_\epsilon. \tag{25}$$

Observe that the analysis presented in this section is valid for both linear and nonlinear phase FIR filters implemented in direct form. Since nonlinear phase FIR filters meet a given magnitude response specification with filter length smaller than that of linear phase FIR filters [24], [25], (25) shows that nonlinear phase filter implementations are less affected by random errors in the filter coefficients. (For a discussion on how much the filter length can be reduced by giving up the phase linearity, see [24].)

III. Simulation Results and Discussion

Extensive simulations have been carried out to verify the analysis described in the last section. In all cases the filter coefficients of the ideal filter have been obtained with the well-known McClellan–Parks algorithm [29]. However, since our analysis is not restricted to the linear phase FIR transfer functions, it is assumed that the actual implementation does not exploit the symmetry in the filter coefficients.

For the frequency responses in Fig. 1, for example, we have $N = 50$, $\delta_2 = 4.467 \times 10^{-6}$ (-107 dB), and $\sigma_\epsilon = 0.001$. In this case we obtain $E\{|\Delta H(\omega)|\} = 6.267 \times 10^{-3}$, which is much larger than δ_2. This result means that *with a very high probability*, an SC transversal filter of length $N = 50$ is not able to achieve the desired stopband attenuation. Observe that for this example, (25) gives $E\{|\hat{H}(\omega)|\} \approx E\{|\Delta H(\omega)|\} = -44$ dB, which is in accordance with the frequency responses in Fig. 1. To verify the accuracy of these results we computed the average of the frequency

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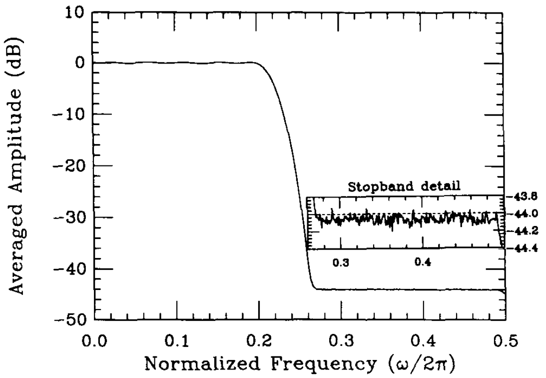

Fig. 4. Average amplitude computed over 10000 filters with specifications as given in Fig. 1. Observe that in accordance with (24), $E\{|\hat{H}(\omega)|\} \leqslant -44$ dB in the stopband.

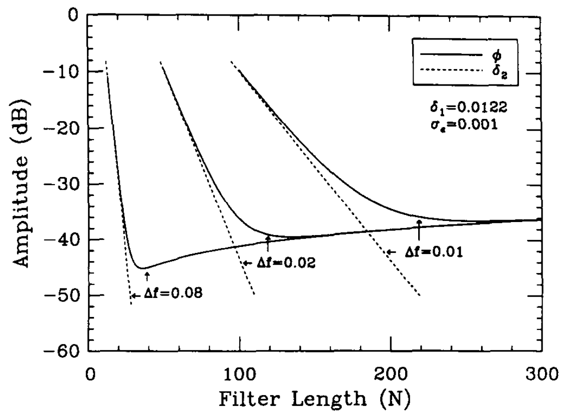

Fig. 5. Upper bounds (solid lines) for expected stopband amplitudes as given by (26).

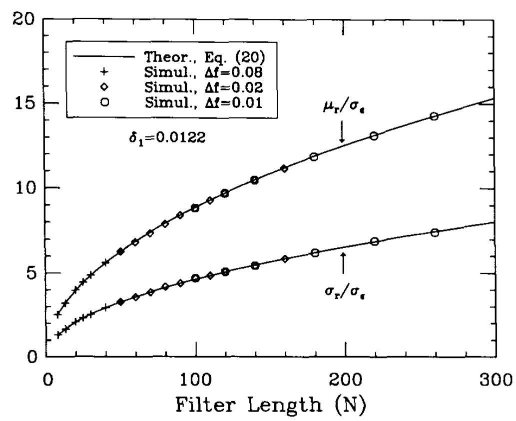

Fig. 6. Simulation results obtained from 10000 FIR filters to verify the expressions given in (20).

response amplitudes of 10000 samples of FIR filters ($N = 50$, $\delta_1 = 0.0122$, and $\Delta f = 0.08$) with Gaussian coefficient errors having $\sigma_\epsilon = 0.001$. The results are displayed in Fig. 4. Notice the perfect agreement between the predicted stopband attenuation (-44 dB, as calculated above) and the computed values, including the extreme frequencies, in which case the average values are a bit lower. This, as mentioned previously, would have been indicated by the more accurate pdf in (16). Fig. 4 also shows that, in accordance with (24), $E\{|\hat{H}(\omega)|\} \leqslant -44$ dB in the stopband.

Equation (24) gives the upper bound

$$\phi = \delta_2 + \frac{\sqrt{\pi N}}{2}\sigma_\epsilon \tag{26}$$

for the expected value of the actual magnitude of $H(\omega)$ in the stopband. Following an approach by Gersho *et al.* [26], we use the empirical formulas given in [28] relating δ_2 to δ_1, Δf and N. Now the upper bound ϕ can be plotted as a function of N for particular values of δ_1, Δf, and σ_ϵ. Fig. 5 shows (solid lines) upper bounds obtained for $\delta_1 = 0.0122$, $\sigma_\epsilon = 0.001$ and three different values of Δf: 0.01, 0.02, and 0.08. These plots are similar in form to the ones reported in [26], except there the plots are upper bounds for the *maximum amplitude* of the error transfer function. Also shown in Fig. 5 (broken lines) are the amplitudes that would be obtained in the ideal case (if the coefficients were implemented with infinite accuracy). These curves clearly indicate the values of N beyond which the effect of coefficient inaccuracy becomes the dominant factor in determining the stopband amplitude. Let us consider, for instance, the curve corresponding to $\Delta f = 0.08$. We can see that for $N < 30$, the inaccuracy effect is not obvious since both the ideal and upper bound curves coincide. For $N > 30$, on the other hand, the inaccuracy effect dominates and the expected amplitude is as given by (25). The minimum point occurs for $N = 36$, which is the filter length that is expected to provide the best stopband attenuation (-45.15 dB) when $\delta_1 = 0.0122$, $\Delta f = 0.08$, and $\sigma_\epsilon = 0.001$. Observe that for each filter with length $N > 36$ there is always another filter with smaller length ($N < 36$) that provides at least the same stopband attenuation. So, for example, a transversal filter of length $N = 60$, which ideally is supposed to provide $\delta_2 = -138.6$ dB, would actually be limited to $\hat{\delta}_2 = -43.27$ dB. Then the same performance (-43.52 dB) could be obtained with another filter satisfying the same specifications and using only $N = 29$ coefficients.

To verify the validity of (19) as an approximation for the exact expression in (16) 10 000 samples of transversal filters with $\sigma_\epsilon = 0.001$ have been generated, from which both the average and standard deviation of $|\Delta H(\omega)|$ have been computed at $\omega = 0.3$. The ideal prototype FIR filters with $\delta_1 = 0.0122$, $\Delta f = 0.01$, 0.02, and 0.08, and several values of N, have been designed using the McClellan–Parks algorithm [29]. Fig. 6 shows plots of μ/σ_ϵ and σ_r/σ_ϵ given in (20) together with values obtained by simulation. Again, the excellent agreement between the predicted and the simulated results is observed.

Since the pdf of the error transfer function is known, it is possible to obtain the probability of having the stopband amplitude larger than a certain level $\hat{\delta}_2 > \delta_2$. This can be used to define the ideal filter specifications in order to guarantee, with a certain probability, that the stopband attenuation never crosses a predefined level $\hat{\delta}_2$. These aspects are investigated in the following section.

IV. Filter Design Considerations

As we have seen from the previous section, the random errors in the filter coefficients cause the actual amplitude in the stopband to be higher than the ideal value δ_2. Let us then suppose that we want to obtain the probability of having the stopband amplitude larger than a certain level $\hat{\delta}_2 > \delta_2$. Then from (21) we have, in the stopband,

$$\begin{aligned}\Pr\left\{|\hat{H}(\omega)| \geqslant \hat{\delta}_2\right\} &\leqslant \Pr\left\{\delta_2 + |\Delta H(\omega)| \geqslant \hat{\delta}_2\right\}\\ &= \Pr\left\{|\Delta H(\omega)| \geqslant \hat{\delta}_2 - \delta_2\right\}. \quad (27)\end{aligned}$$

Finally, using (19), we obtain

$$\begin{aligned}\Pr\left\{|\hat{H}(\omega)| \geqslant \hat{\delta}_2\right\} &\leqslant \int_{\hat{\delta}_2 - \delta_2}^{\infty} f_R(r)\,dr\\ &= \exp\left[-\frac{\left(\hat{\delta}_2 - \delta_2\right)^2}{N\sigma_\epsilon^2}\right]. \quad (28)\end{aligned}$$

To illustrate the usefulness of such a result, let us use once more the example $N = 50$, $\delta_1 = 0.0122$, $\delta_2 = 4.467 \times 10^{-6}$, and $\Delta f = 0.08$. Then the probability of obtaining a filter with $\hat{\delta}_2 \geqslant -40$ dB is $\Pr\{|\hat{H}(\omega)| \geqslant 40 \text{ dB}\} \leqslant 0.135$. This means that less than 13.5% of the filters built have, in the stopband, a frequency response with magnitude crossing the line of 40 dB of attenuation, provided that the filter coefficients are implemented with $\sigma_\epsilon = 0.001$.

We can also define the maximum standard deviation $\sigma_{\epsilon,\max}$ that guarantees, with a certain probability p, that the stopband amplitude never crosses a predefined level $\hat{\delta}_2$. This is readily obtained from (28) as

$$\sigma_{\epsilon,\max} = \frac{\hat{\delta}_2 - \delta_2}{\sqrt{N \ln(1/p)}}. \quad (29)$$

So, in order to have less than 1% of the filter frequency responses in the previous example crossing the level $\hat{\delta}_2 = -40$ dB, it would be necessary that $\sigma_{\epsilon,\max} = 6.59 \times 10^{-4}$, which is at the limit of the achievable precision of the state-of-the-art technology. Notice that this value is roughly equivalent to 11 bits of precision, easily satisfied in a digital implementation.

As a second example, we consider the design of a low-pass telephone filter [30] with a stopband amplitude $\delta_2 = -40$ dB, passband ripple $\delta_1 = 0.5$ dB, passband and stopband edge frequencies of 3.4 kHz and 4.6 kHz, respectively, and clock frequency of 64 kHz. Using the algorithm in [29] we find that in the absence of coefficient errors, the minimum filter order that would satisfy such requirements is $N = 92$. In this case, with $\sigma_\epsilon = 0.001$, the expected magnitude of the error transfer function is $E\{|\Delta(\omega)|\} = -41.4$ dB, which is comparable to the desired stopband amplitude. Using (28) we obtain $\Pr\{|\hat{H}(\omega)| \geqslant -35 \text{ dB}\} \leqslant 0.518$, and from (29) it follows that it is necessary to have $\sigma_{\epsilon,\max} = 3.78 \times 10^{-4}$ to stay below -35 dB in the stopband with a probability of more than 99%.

V. Concluding Remarks

An analysis of the distortion introduced by capacitance ratio inaccuracies in the frequency response of FIR SC filter implemented in direct form has been presented. Assuming that the coefficient errors have identical Gaussian distribution, an exact pdf has been derived for the error in the frequency response. It has been shown that this distribution can be characterized by a Rayleigh distribution, which has been used to derive an upper bound for the expected stopband attenuation. The results of the analysis can be used to predict, for a given implementation technology for which the capacitance ratio accuracy is known, the achievable stopband attenuation. Simulation results have been shown to support the analysis.

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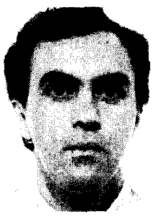

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