Basic Multirate Building Blocks

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Introductory Remarks

Multirate Signal Processing

- Substantial knowledge has been developed in digital multirate signal processing, specially in the early 1980s, when digital signal processing algorithms were becoming increasingly powerful;
- By that time, advances in IC technologies were making it possible the realization of DSPs capable of performing billions of operations per second;
- The motivation was to adjust the signal sampling rate according to its bandwidths variations from the input to the output of the network;
- Efficient structures have been derived to exploit this idea, as for example, the celebrated subband coding scheme, by which the signal is split into a number of frequency bands, and appropriately coded inside each band.

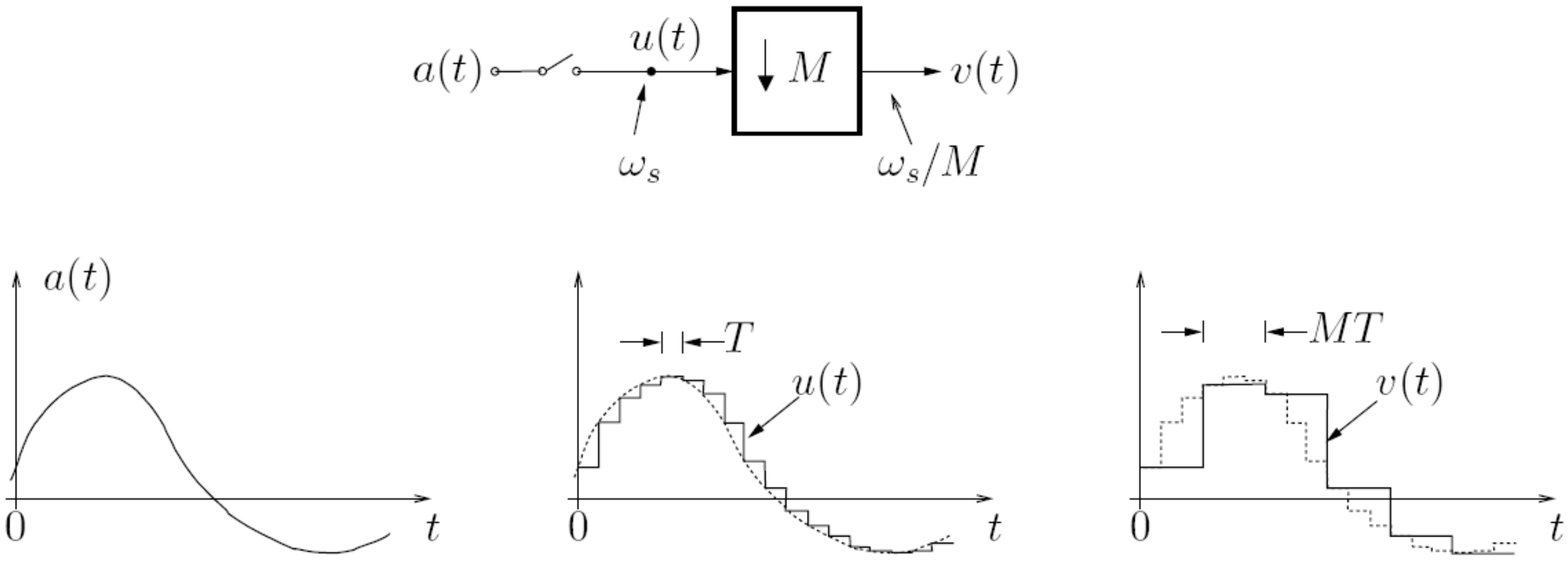
Introductory Remarks

Analog Multirate Signal Processing

- The multirate approach is particularly important for analog networks to relax amplifier settling time requirements and reduce capacitance spread, thereby reducing power consumption and chip area;
- High quality CMOS capacitances and switches have made switched-capacitor (SC) networks capable of incorporating the multirate techniques originally developed in digital signal processing;
- With the multirate option another degree of freedom can be obtained in the design of analog networks.

Building Blocks

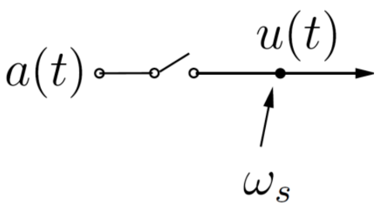
Analog Down-Sampler



M is a positive integer greater than 1.

Building Blocks

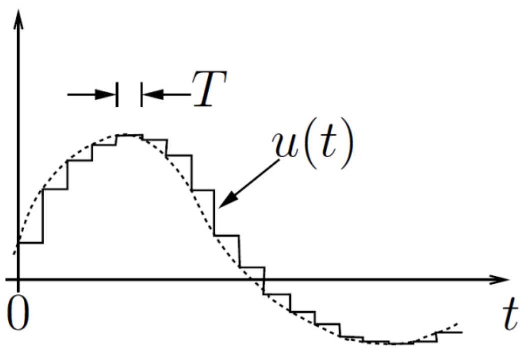
Sample-And-Hold Effect

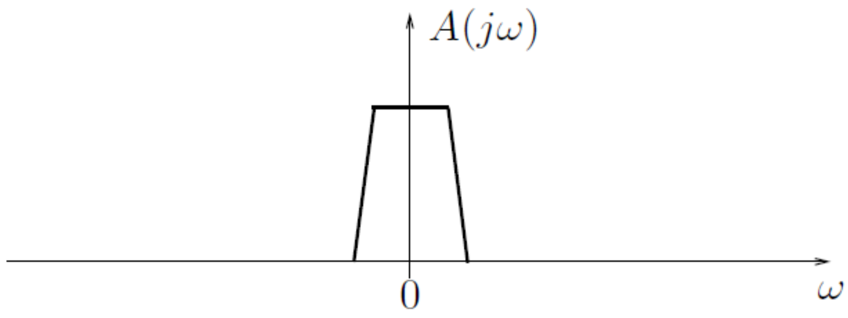


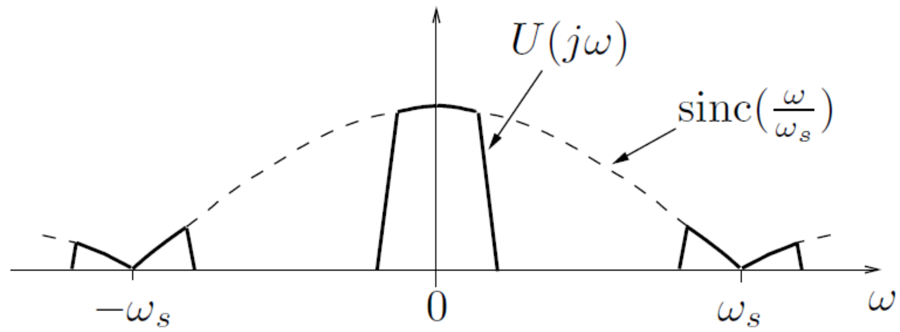
$$U(j\omega) = X(j\omega)S(j\omega)$$

$$X(j\omega) = \frac{\omega_s}{2\pi}\sum_{k=-\infty}^{\infty} A(j\omega - jk\omega_s)$$

$$S(j\omega) = \frac{2\pi}{\omega_s}\,\mathrm{sinc}\left(\frac{\omega}{\omega_s}\right)e^{-j\pi\omega/\omega_s}$$

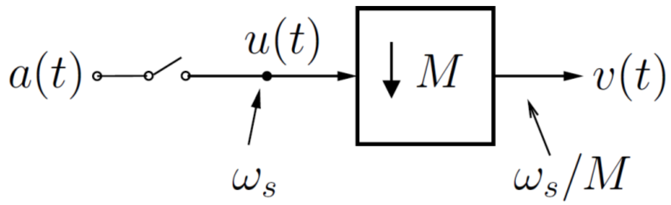


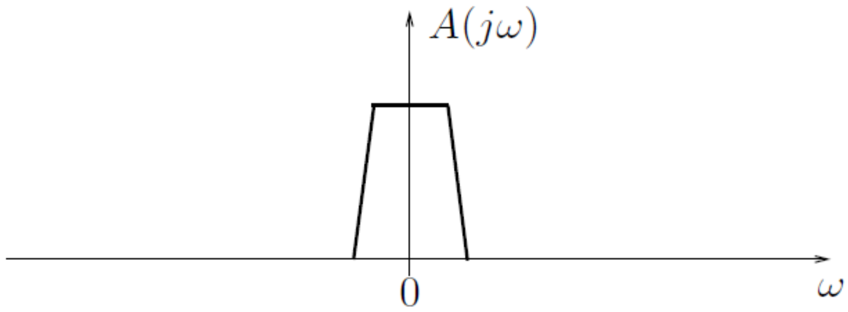


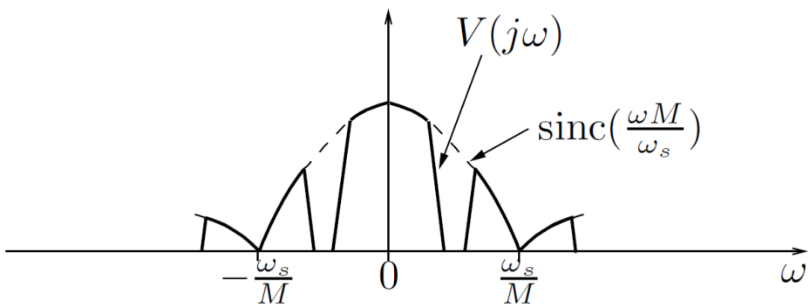


Building Blocks

Sample-And-Hold Effect







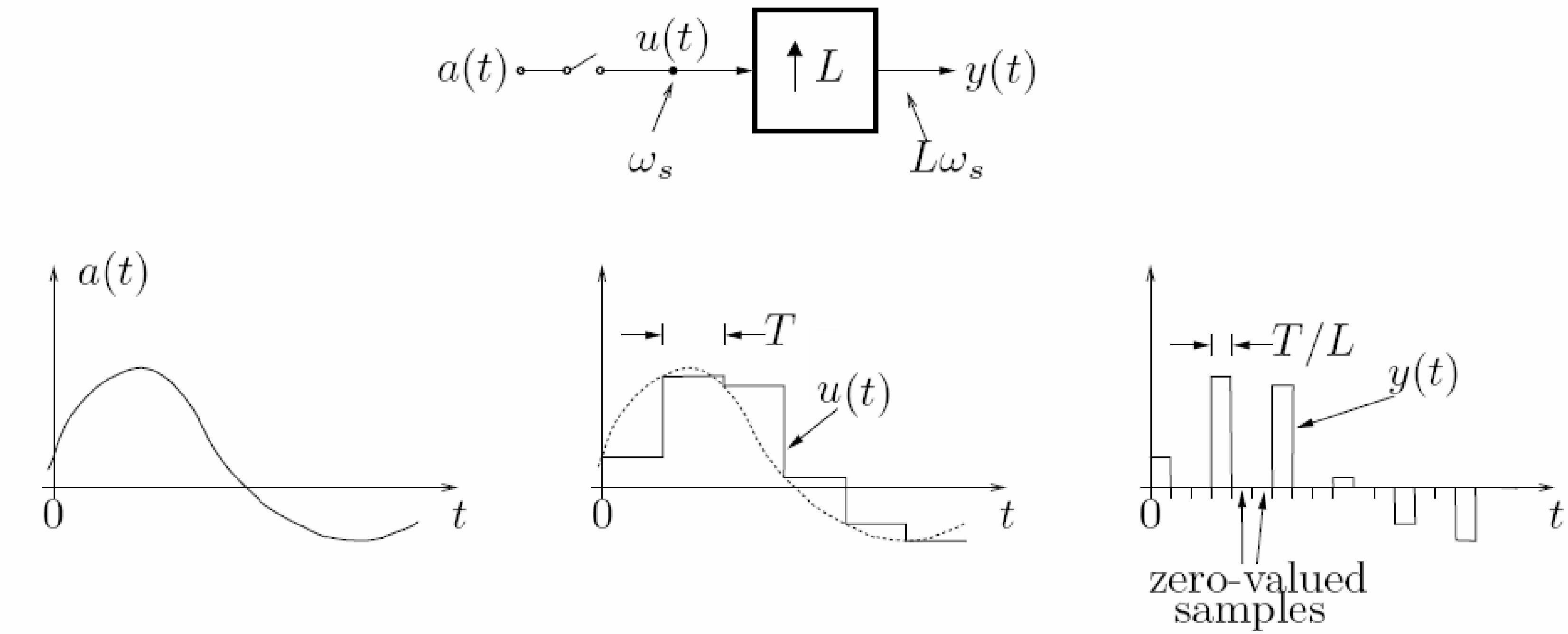
$$V(j\omega) = \left(\frac{\omega_s}{2\pi M} \sum_{k=-\infty}^{\infty} A\left(j\omega - j\frac{k\omega_s}{M}\right)\right) S'(j\omega)$$

$$S'(j\omega) = \frac{2\pi M}{\omega_s} \operatorname{sinc}\left(\frac{\omega M}{\omega_s}\right) e^{-j\pi\omega M/\omega_s}$$

– The down sampler must be preceded by a sampled-data lowpass filter with cutoff frequency $\omega_S/2M$ to prevent aliasing.

Building Blocks

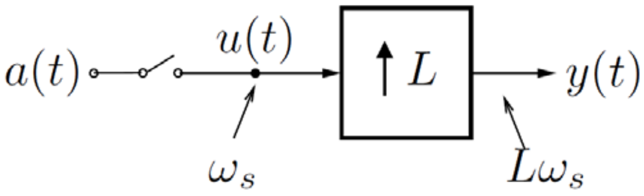
Analog Up-Sampler



- Narrows every sample by factor L, leaving L – 1 zero-valued samples between every 2 adjacent samples.

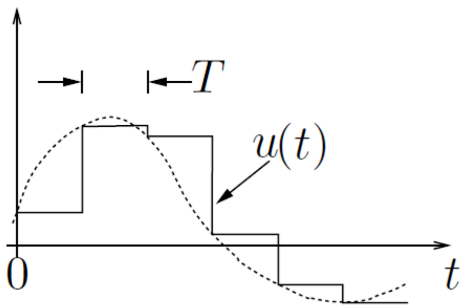
Building Blocks

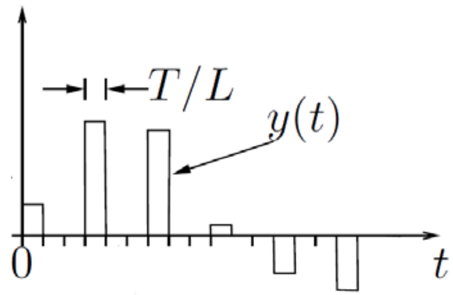
Sample-And-Hold Effect

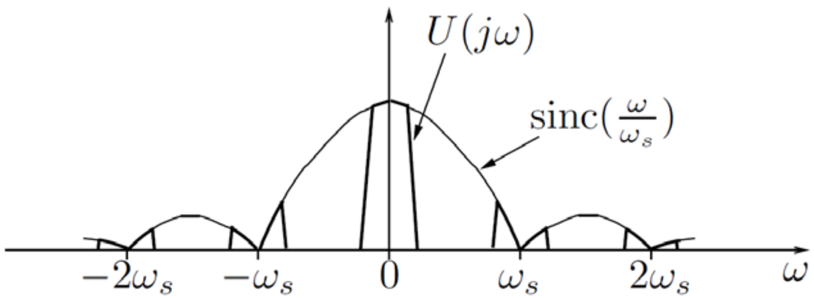


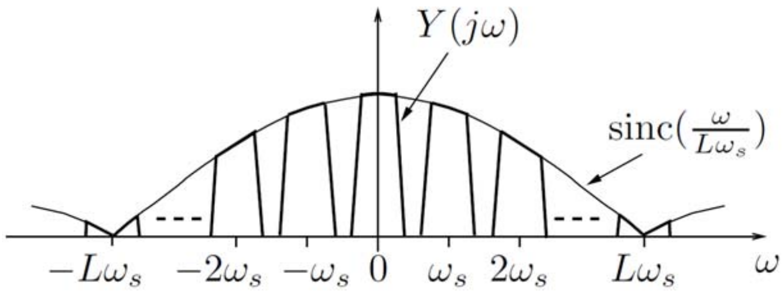
$$Y(j\omega) = \left(\frac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} A(j\omega - jk\omega_s) \right) S''(j\omega)$$

$$S''(\omega) = \frac{2\pi}{L\omega_s} \operatorname{sinc}\left(\frac{\omega}{L\omega_s}\right) e^{-j\pi\omega/L\omega_s}$$



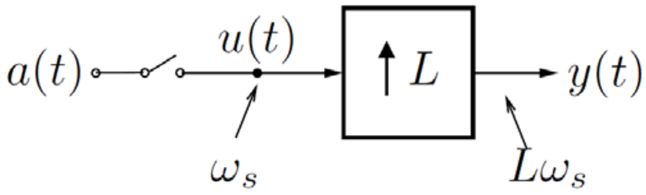


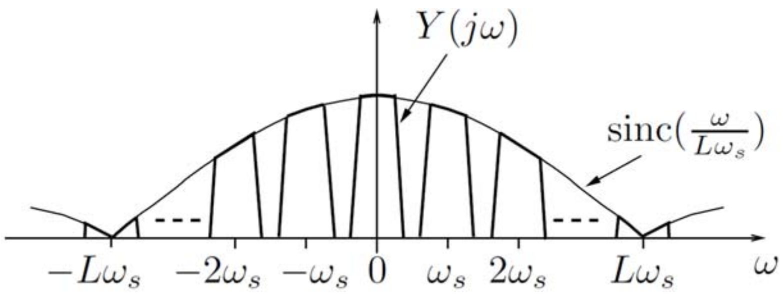




Building Blocks

Sample-And-Hold Effect



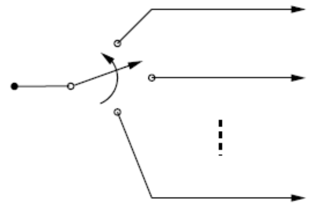


- The $L - 1$ images of the input spectrum carry redundant information;

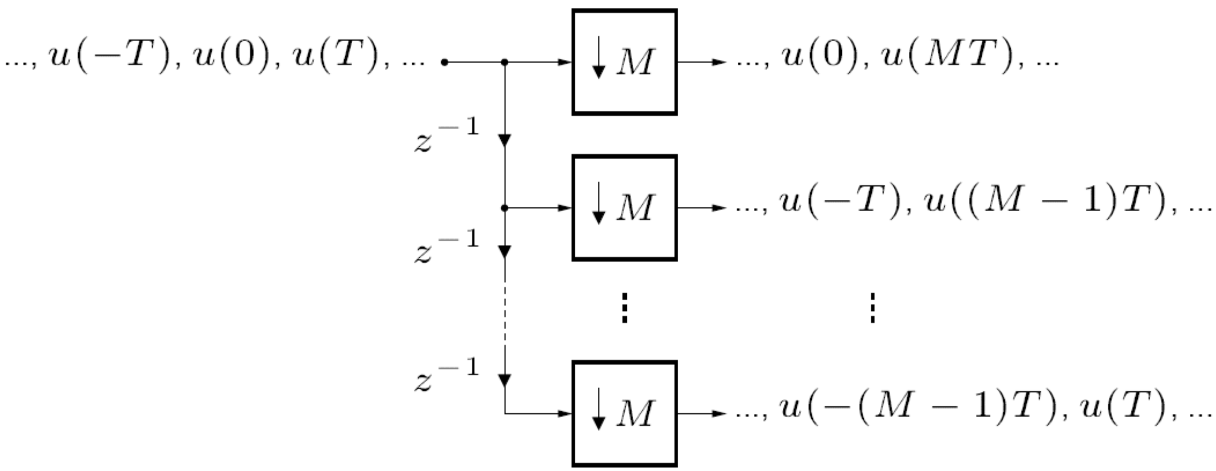
- Can be removed by a sampled-data lowpass filter with cutoff frequency $\omega_S/2$.

Multirate Equivalences

Demultiplexer



(a)

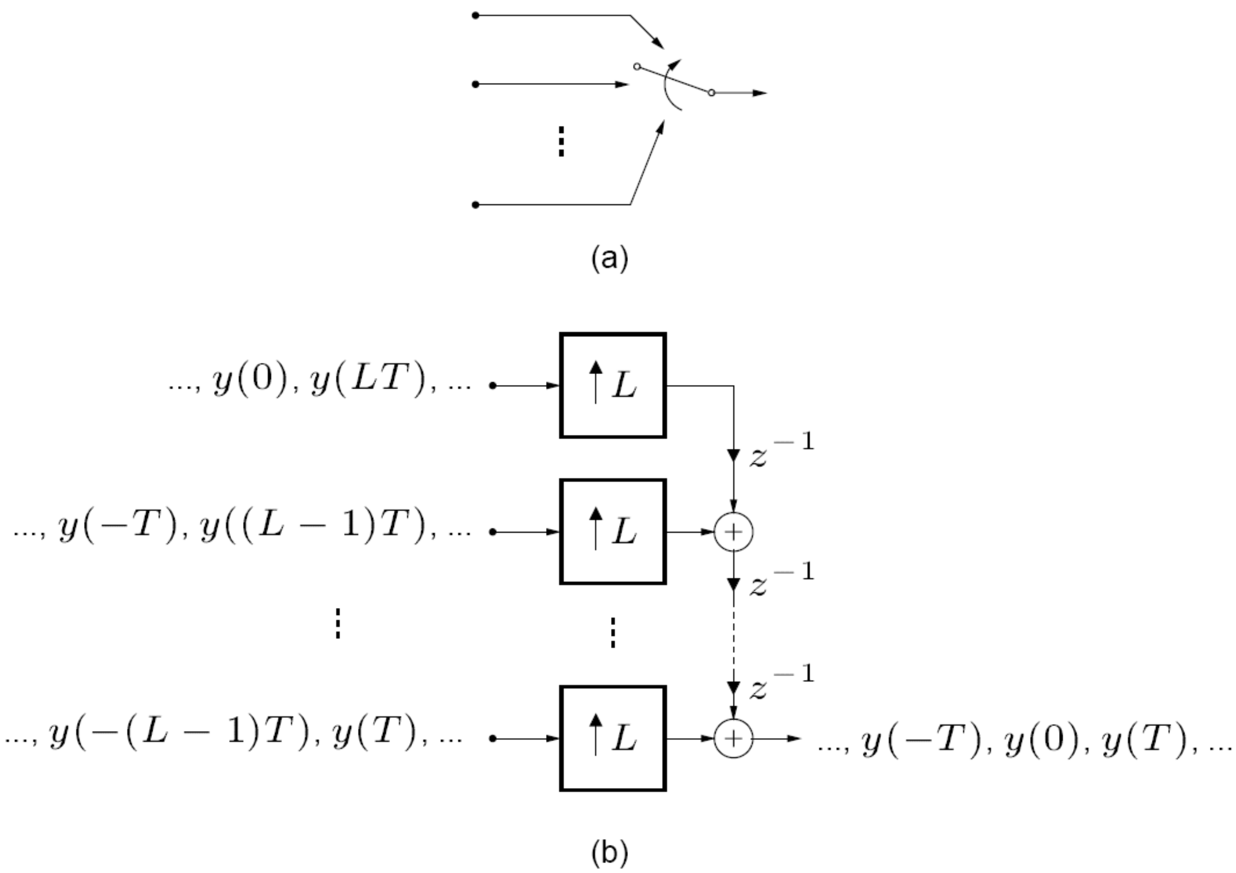


(b)

(a) Functional diagram; (b) Equivalent block diagram.

Multirate Equivalences

Multiplexer



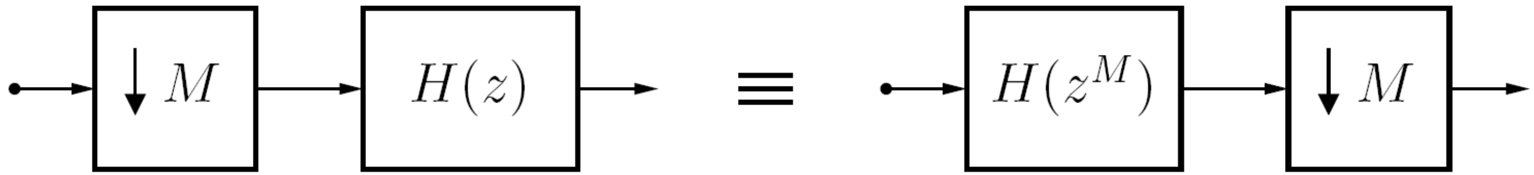
(a) Functional diagram; (b) Equivalent block diagram.

Multirate Equivalences

The Nobel Identities

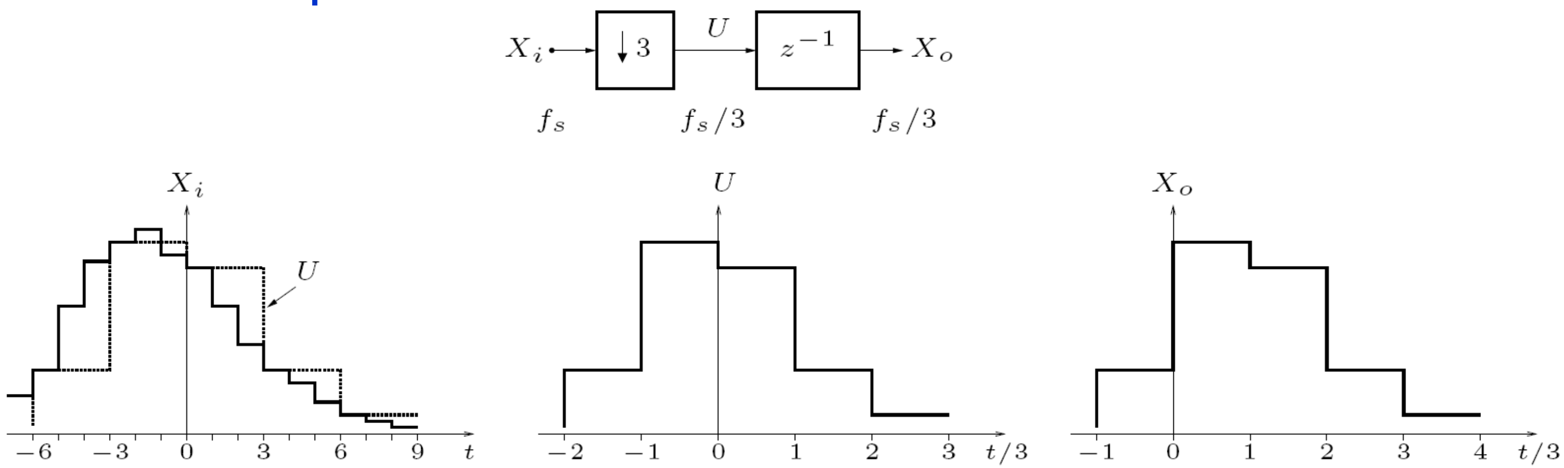
- Often lead to efficient realizations of multirate systems;
- Enable us to “move around” certain building blocks;
- Useful in design and analysis of multirate systems.

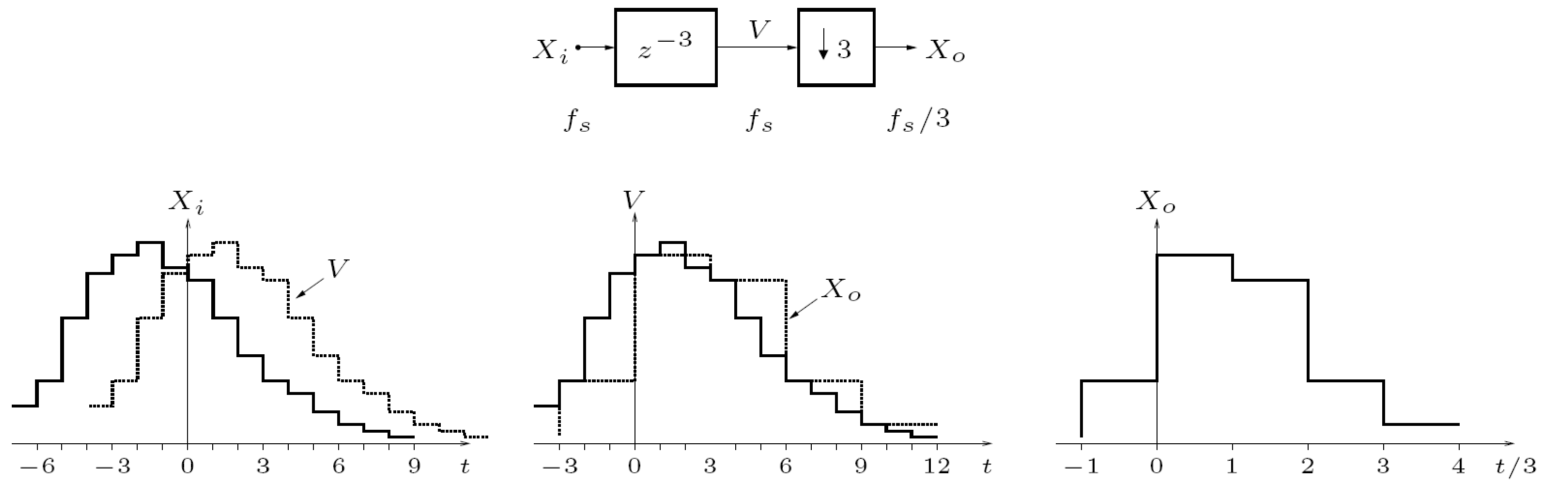
(I) For Down-samplers



Multirate Equivalences

Illustrative Example



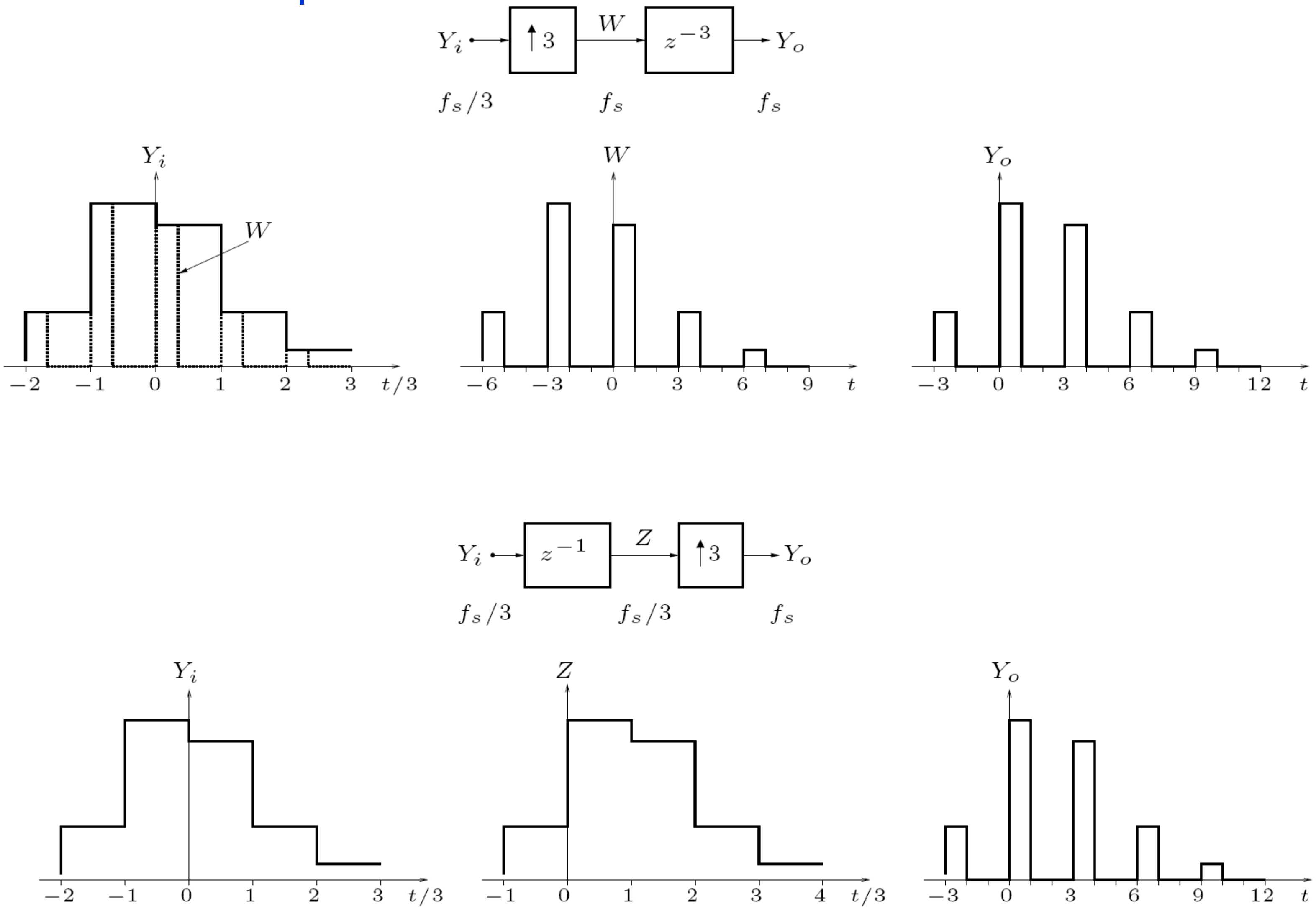


Multirate Equivalences

(II) For Up-samplers

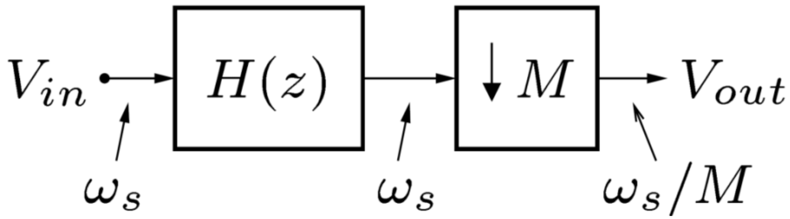
Multirate Equivalences

Illustrative Example



Decimation and Interpolation Filters

Decimation Filter



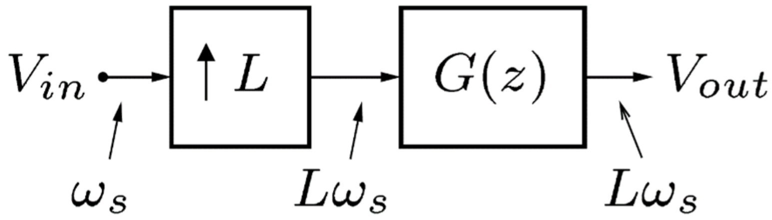
Ideal Frequency Response

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega_s/2M \\ 0, & \omega_s/2M < |\omega| \le \omega_s/2 \end{cases}$$

- The cascade of a lowpass filter and a down-sampler is called a *decimation filter*;
- Filter *H*(*z*) attenuates input frequency components greater than $\omega_s/2M$;
- The sampling rate can then be reduced by the factor *M*.

Decimation and Interpolation Filters

Interpolation Filter

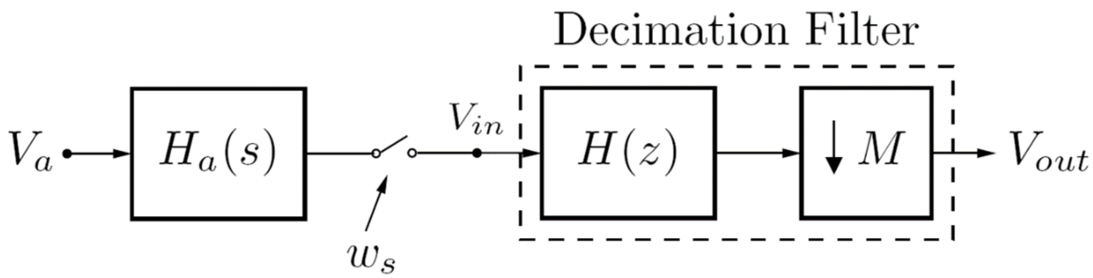


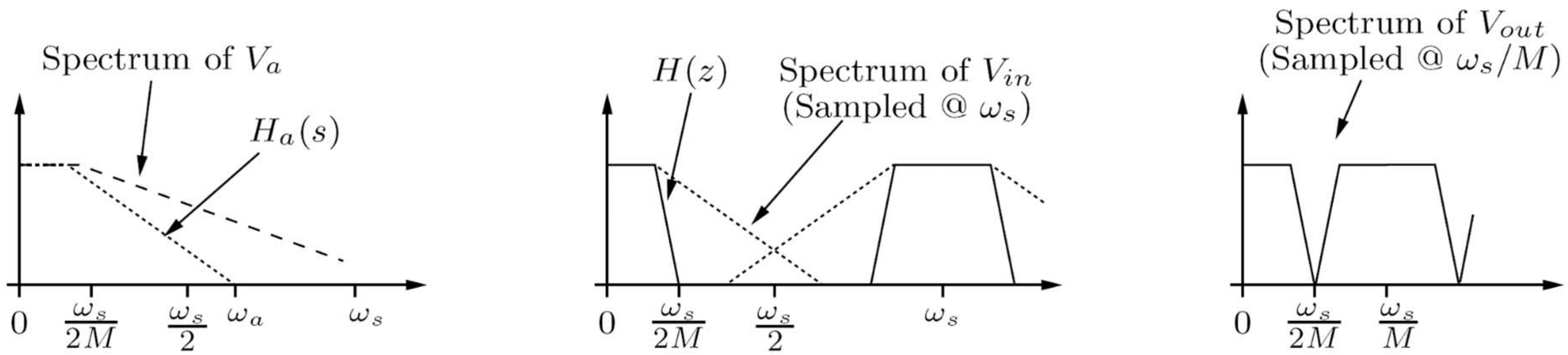
Ideal Frequency Response

$$G(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \omega_s/2 \\ 0, & \omega_s/2 < |\omega| \leq L\omega_s/2 \end{cases}$$

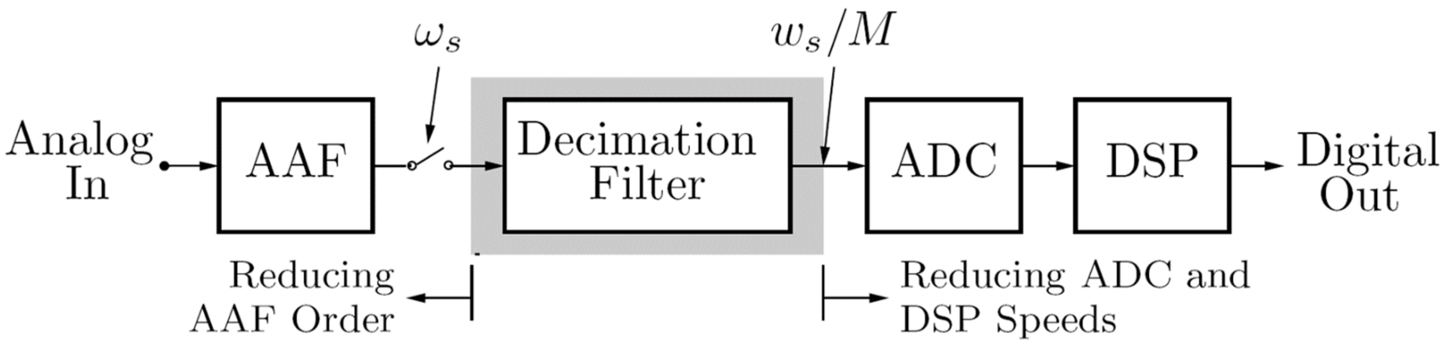
- The cascade association of an up-sampler with a lowpass filter is called an *interpolation filter*;
- The zero-valued samples are filled in by the lowpass filter, hence implementing interpolation;
- In the frequency domain $L - 1$ copies of the input spectrum are created between consecutive multiples of the input sampling frequency.

Sampling of Analog Signals



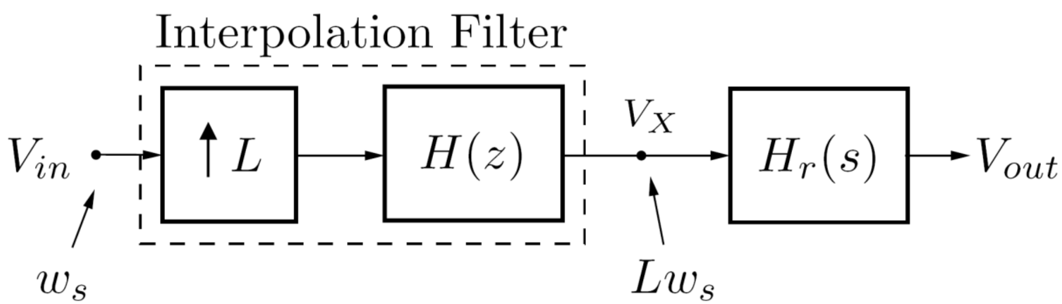


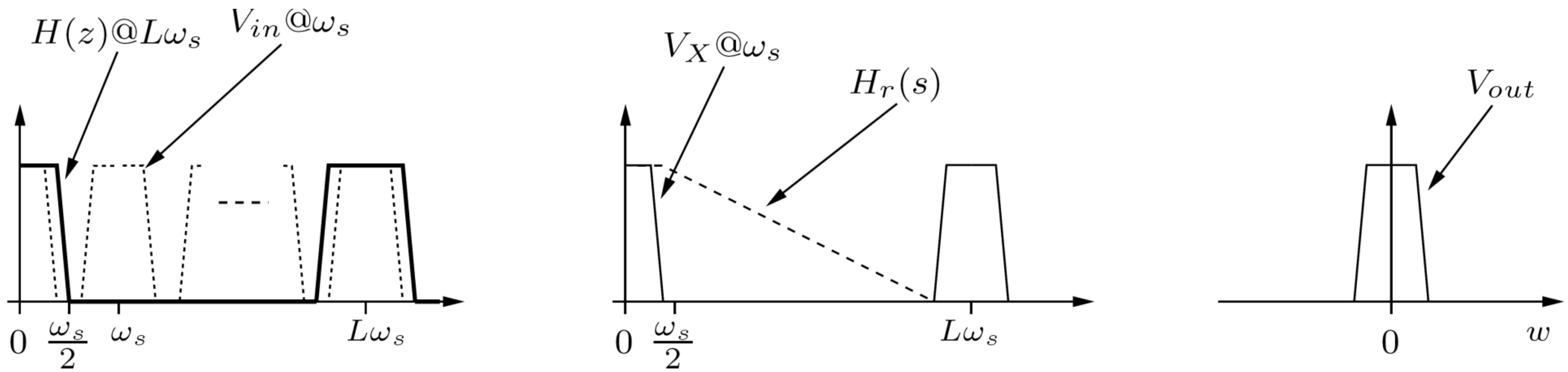
Sampling of Analog Signals



- Output of the anti-aliasing filter (AAF) is oversampled by factor *M*:
  → transition band of AAF can be increased;
- Accurate bandlimiting provided by SC filter H(z) allows sampling rate reduction:
  → reduction of ADC and DSP power consumption.

Reconstruction of Analog Signals





- Interpolation filters are useful in alleviating the specifications of reconstruction filters, $H_r(s)$, that follow D/A converters;
- The SC filter $H(z)$ removes the $L - 1$ images produced by the up-sampler;
- $H_r(s)$ has thus the easier task of selecting (smoothing) the baseband components of V_{in} .