High-Speed A/D Conversion

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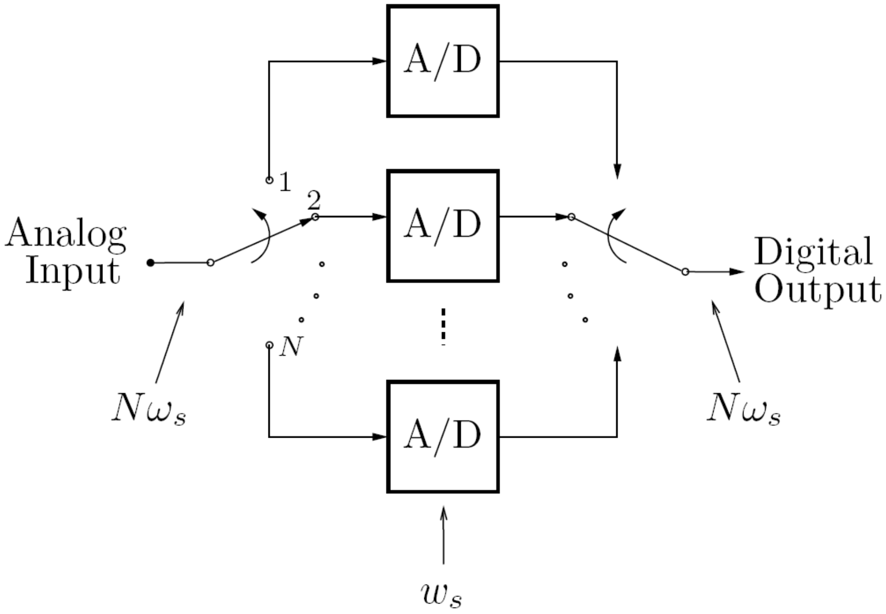
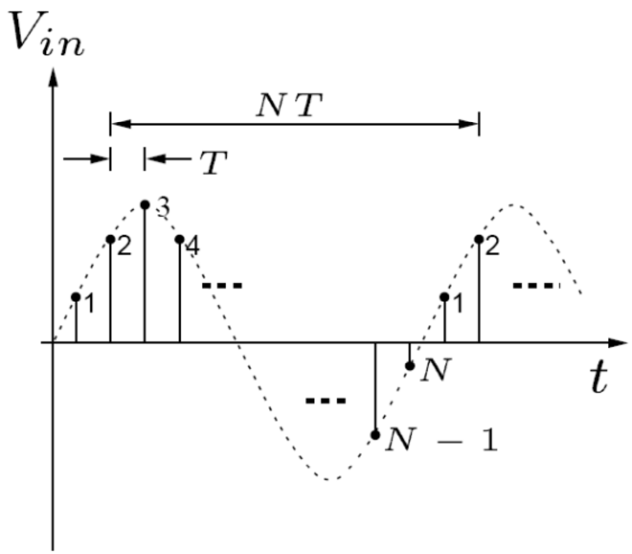
Organization

- **Time-Interleaved ADCs**
  - Operation
  - Jitter, offset and mismatch effects

- **Hybrid Filter Banks**
  - Definition of perfect reconstruction, amplitude reconstruction and phase reconstruction HFBs
  - Application in high-speed ADCs
  - Attenuation of aliasing components

Time Interleaved A/D Conversion





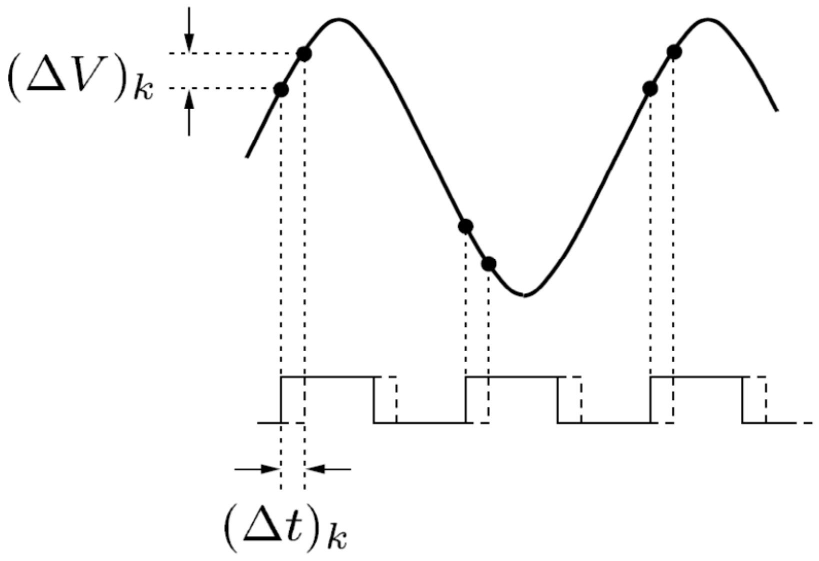
- Ideally, the conversion speed increases as the number of subconverters increases, whereas the resolution of the entire converter equals that of the subconverters;

- In practice, however, mismatches among the subconverters and uneven sampling may seriously degrade its performance;

- On the other hand, compared to a flash converter, its main competitor, advantages in both area and power consumption can be obtained if careful design and layout are considered.

Time Interleaved A/D Conversion

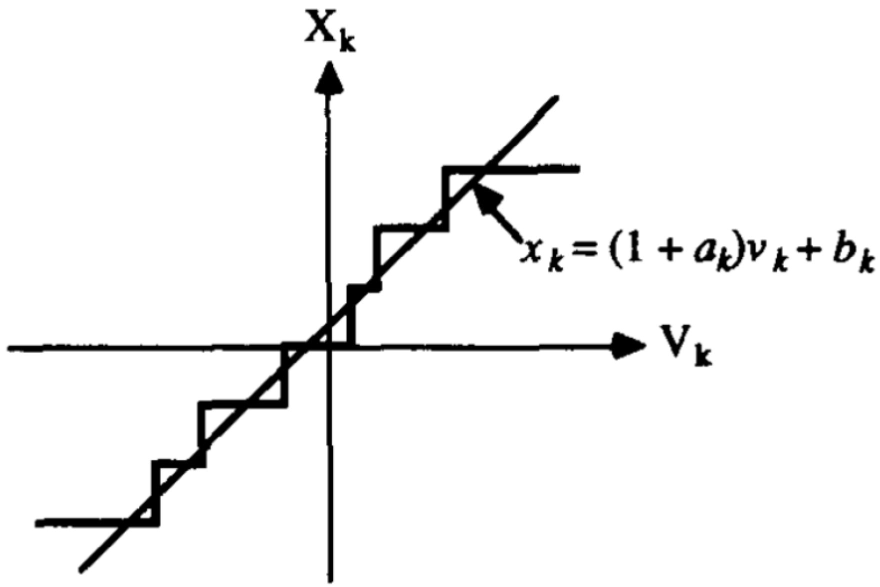
Practical Difficulties



- Uneven sample timing of the input analog signal is caused by clock skew (systematic error) and clock jitter (random error);
- For Gaussian (0,σ) random sample timing errors Δt and a sinusoidal input with frequency ω_o, a decrease of 1 bit of resolution results from a 2-fold increase of either ω_o or σ.

Time Interleaved A/D Conversion

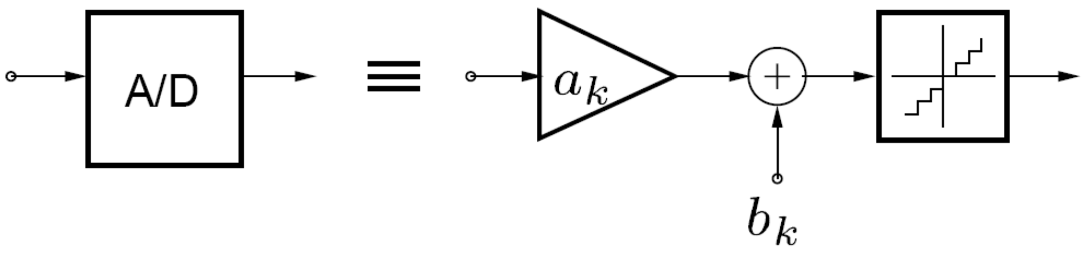
Practical Difficulties



Model of the k-th subconverter

Time Interleaved A/D Conversion

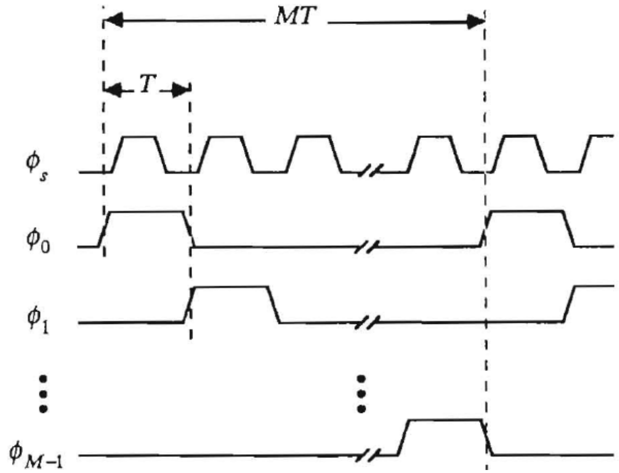
Practical Difficulties

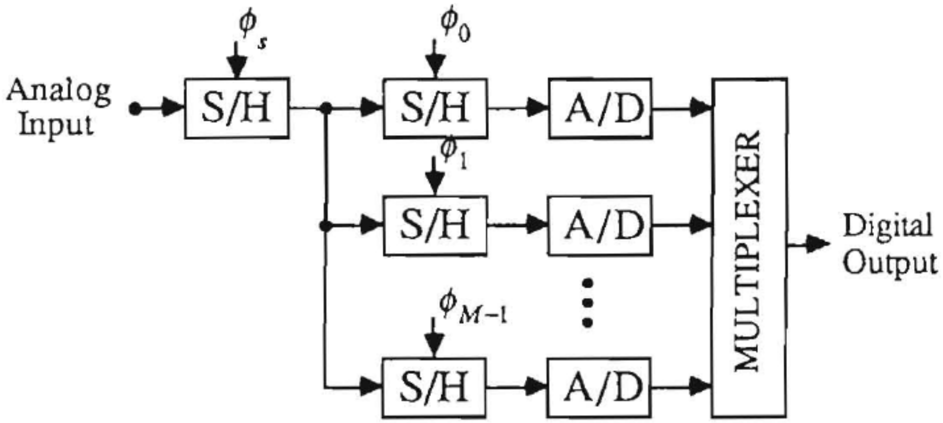


- Distortion caused by gain (a_k) and offset errors (b_k);
- Offset mismatches produce tones at multiples of the lower sampling frequency, that is, at $n\omega_S/N$, n = 1, 2, ..., N - 1, independently of the input amplitude and frequency;
- Gain mismatches produce aliasing distortion in the resulting digital output;
- For Gaussian $(0,\sigma_a)$ random gain errors, 1 bit of resolution is lost *per* 2-fold increase of σ_a.

Time Interleaved A/D Conversion

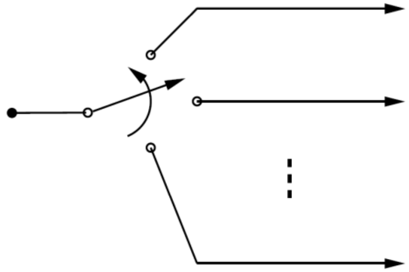
- Despite the above drawbacks, time-interleaved A/D converters have received increased attention for many years;
- A large variety of digital calibration techniques have been applied to compensate mismatch errors effects;
- A so-called *two-rank* demultiplexing scheme has also been proposed to reduce jitter effects.

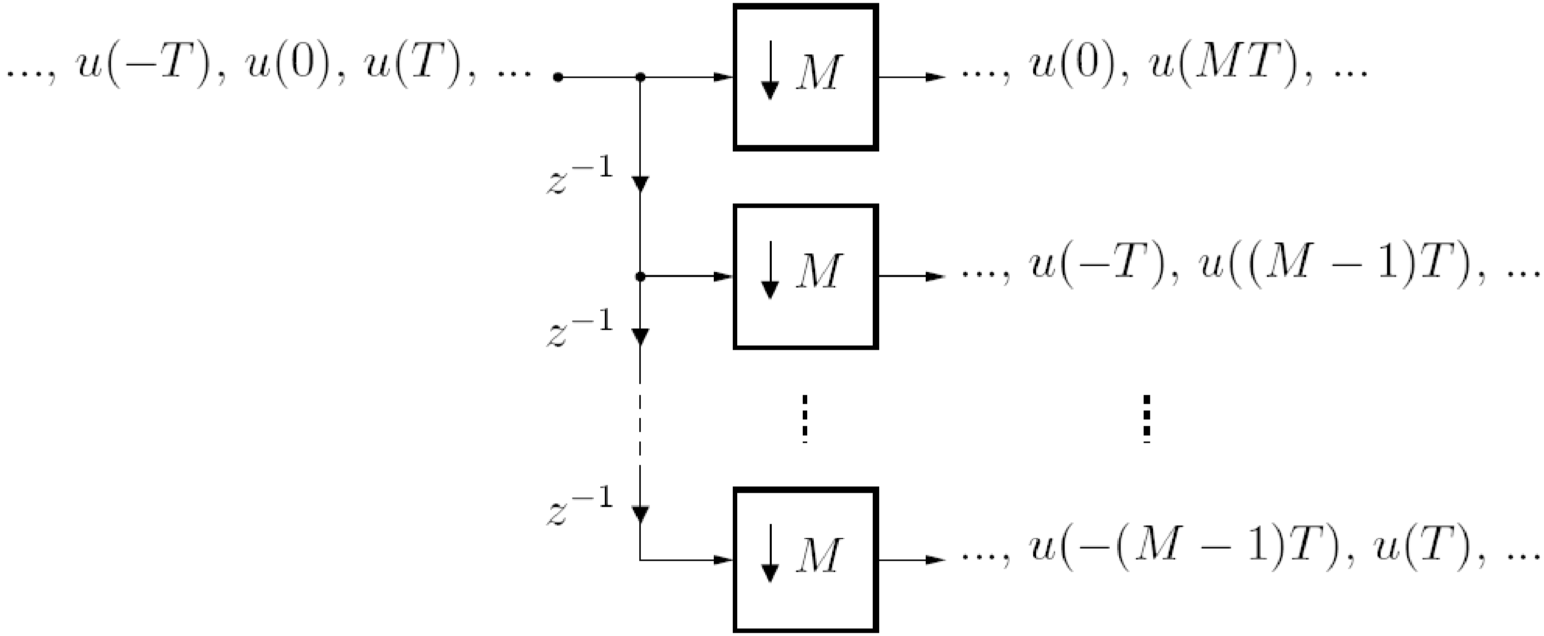




Multirate Equivalences

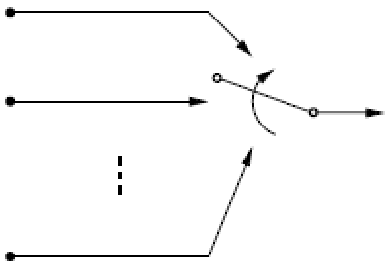
Demultiplexer

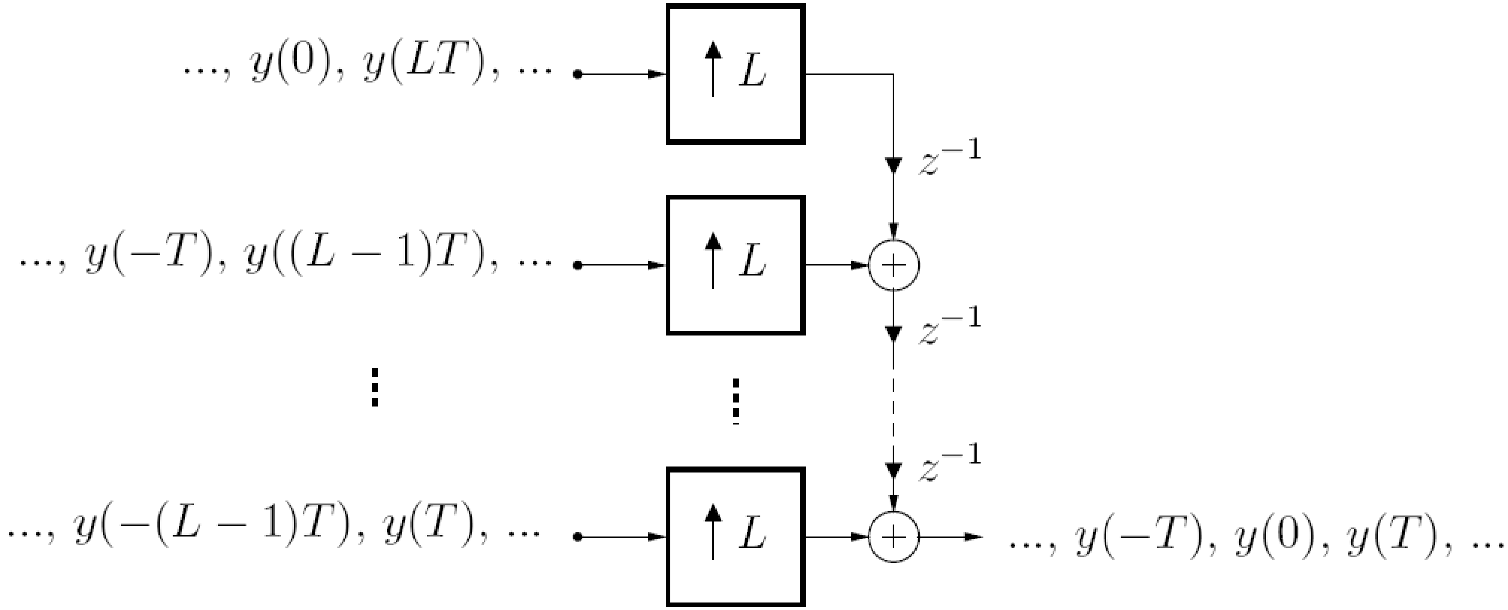




Multirate Equivalences

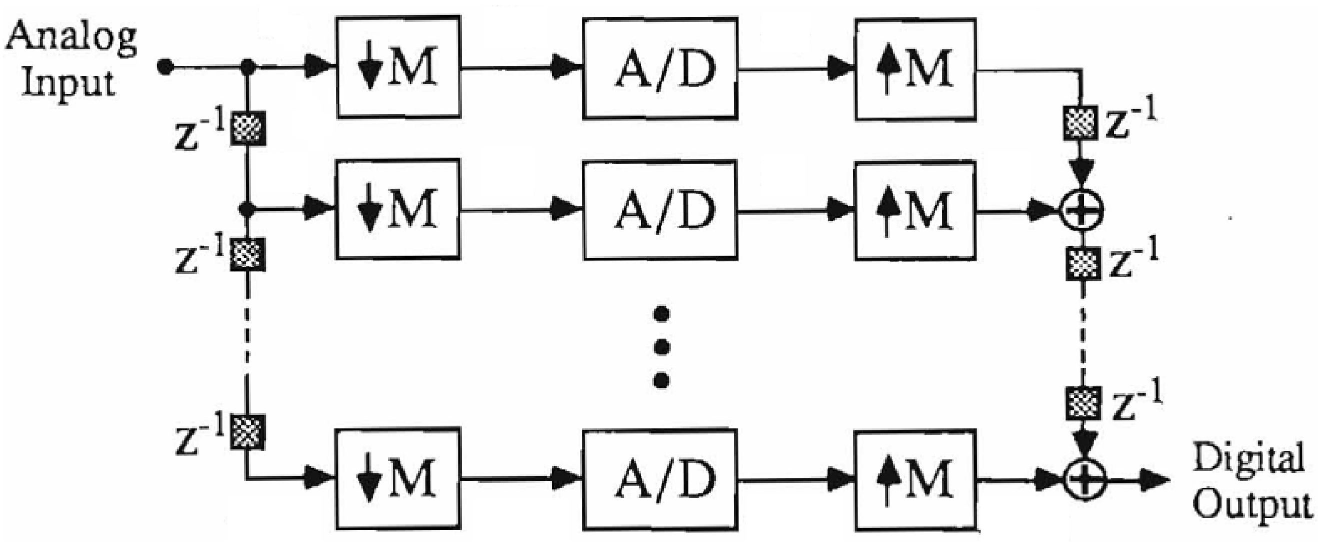
Multiplexer





Multirate Equivalences

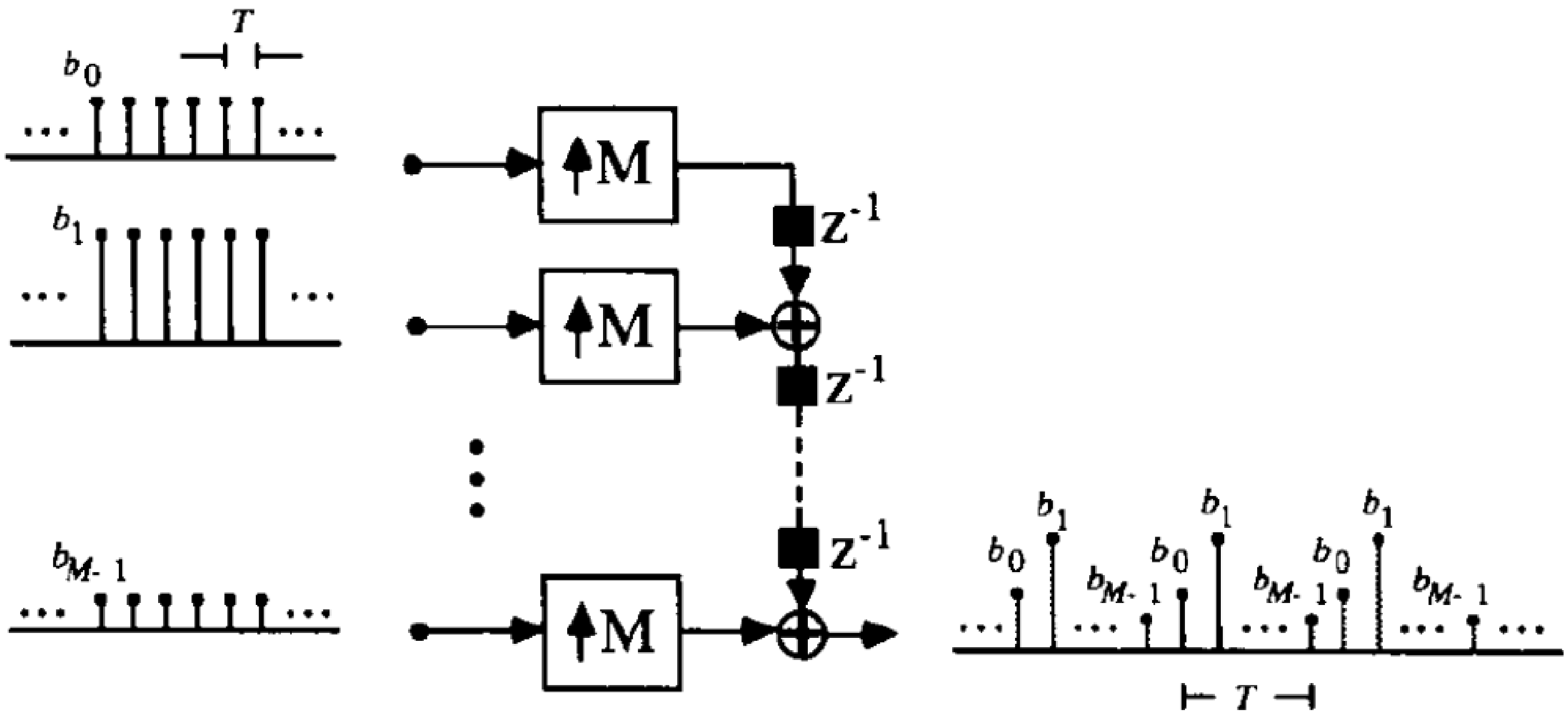
Time-interleaved A/D Converter



Multirate modelling – suitable for mathematical error analysis.

Multirate Equivalences

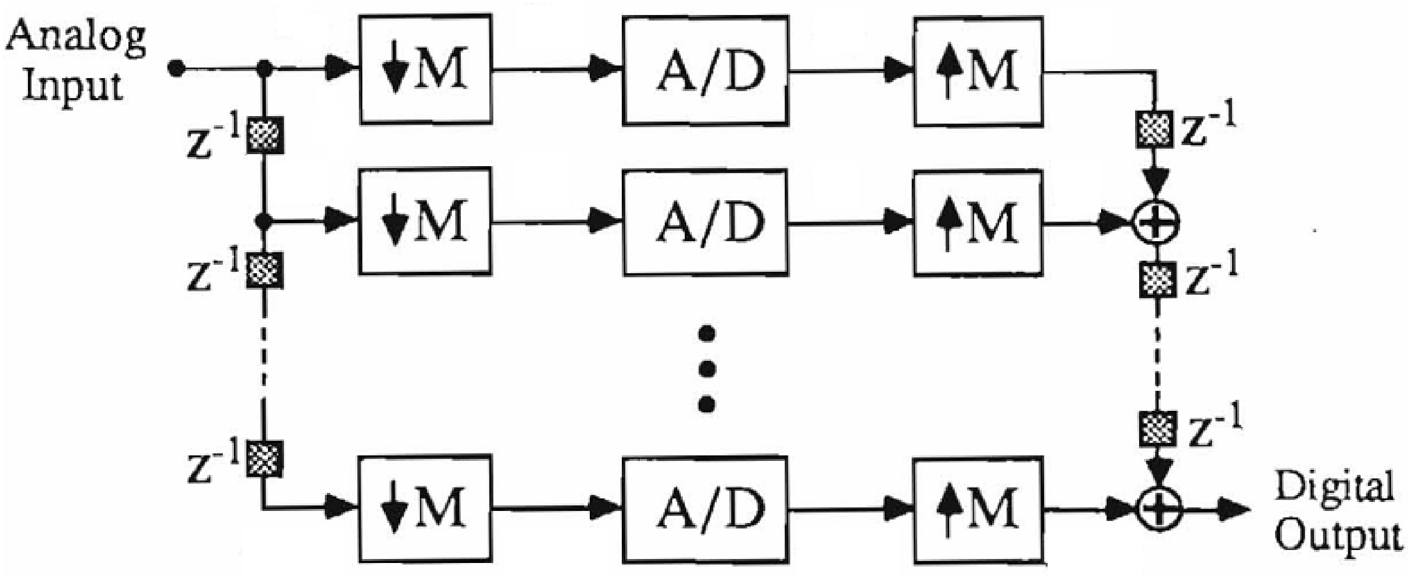
Time-interleaved A/D Converter



Offset mismatches: produce tones at multiples of the input sampling frequency, $n\omega_S/M$, n = 1, 2, ..., M - 1, independently of the input amplitude and frequency;

Multirate Equivalences

Time-interleaved A/D Converter

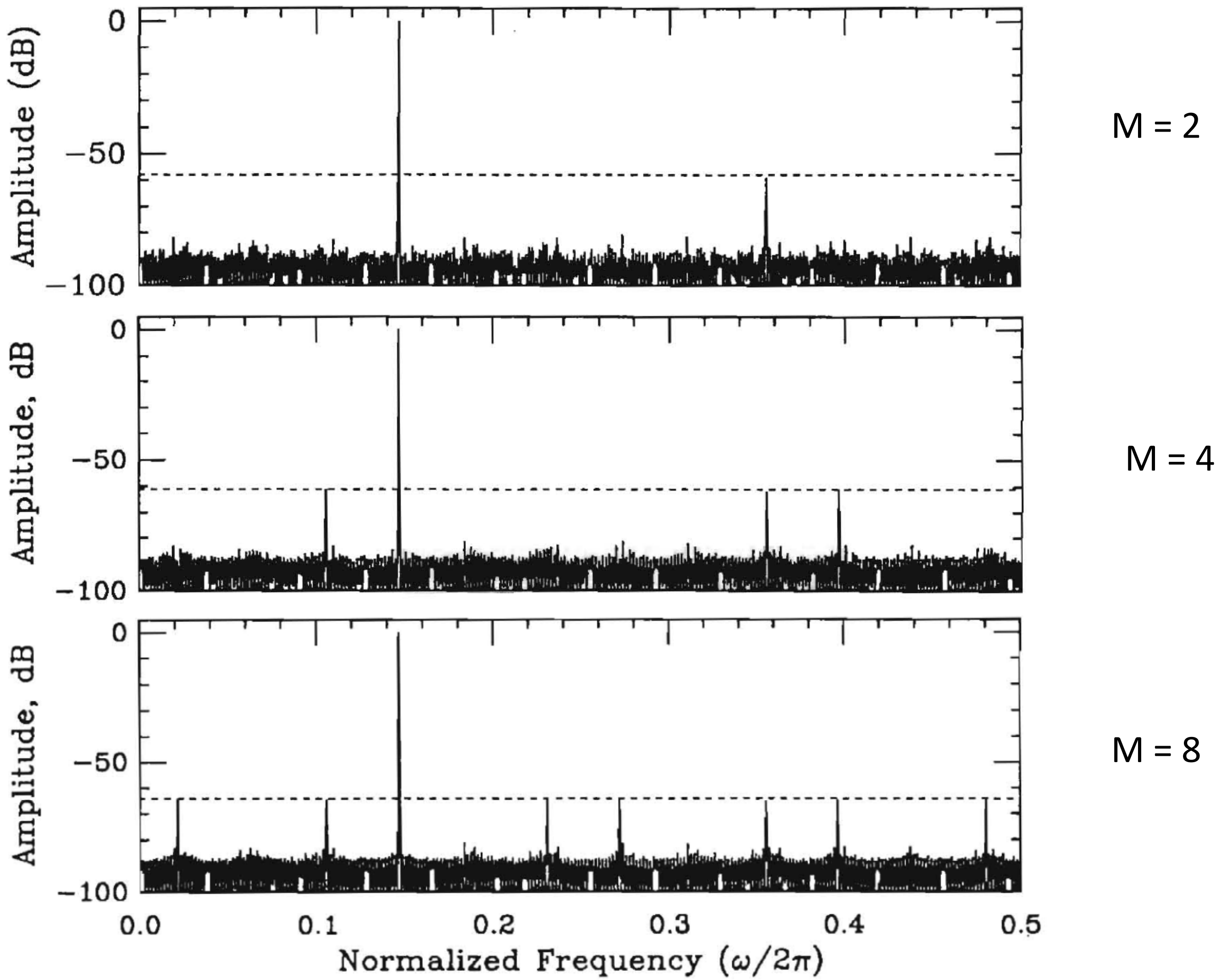


Gain mismatches: produce aliasing; the magnitude of each aliased component of the input signal is scaled by $\pi\sigma_a/4M$.

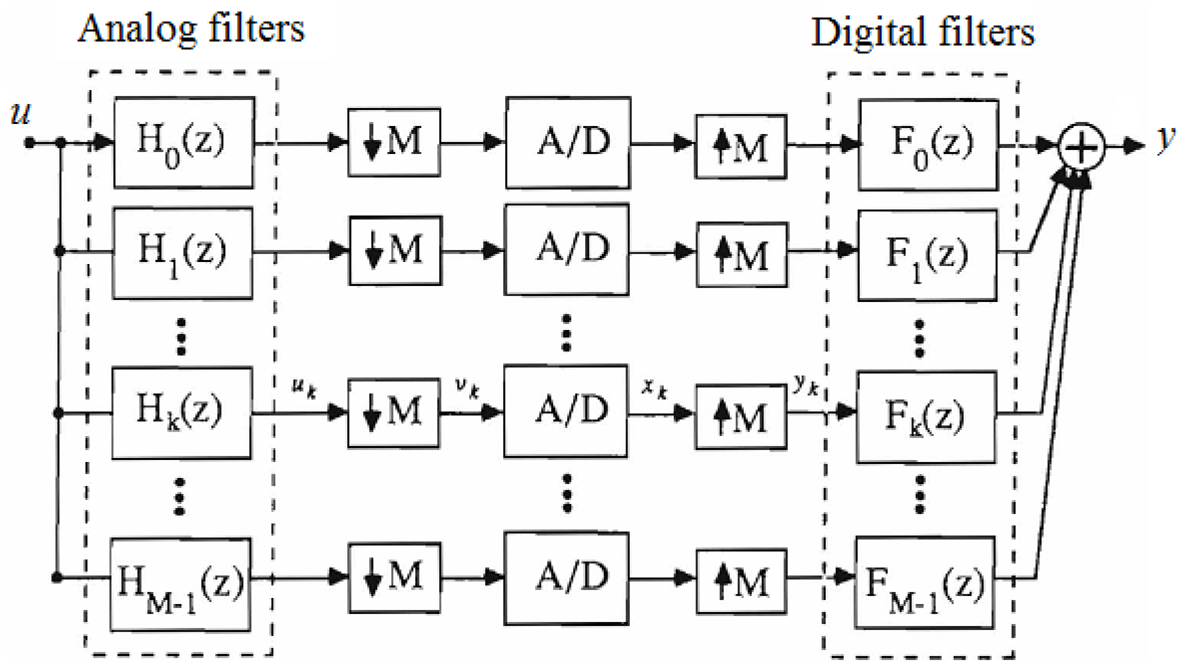
- Example: σ_a = 0.01, M = 4 ⇨ Each of the 3 aliased components are expected to have magnitudes of – 47.3 dB.

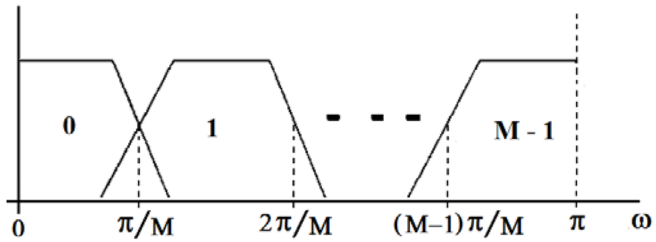
Multirate Equivalences

Ex.: 9-bit subconverters; σ_a = 0.002; 1024-point FFTs; ω_o = 149/1024;



Hybrid Filter Banks





- If $F_k(z) = z^{-(M-1-k)}$ and $H_k(z) = z^{-k}$ ⇨ Time-interleaved A/D;
- The analog (SC) filters followed by the down-sampler array act as the *two-rank* S/H technique.

Hybrid Filter Banks

Frequency Domain Relationships

$$V(z) = \frac{1}{M}\sum_{l=0}^{M-1} U(z^{1/M}W^{l}), \quad W = e^{-j2\pi/M}$$

$$Y(z) = X(z^{M})$$

Hybrid Filter Banks

k-th branch:

Neglecting quantization errors ⇨ $x_k(nT) = v_k(nT)$

$$Y(e^{j\omega}) = \frac{1}{M}\sum_{\ell=0}^{M-1} U\left(e^{j(\omega+\frac{2\pi\ell}{M})}\right)\sum_{k=0}^{M-1} H_k\left(e^{j(\omega+\frac{2\pi\ell}{M})}\right)F_k(e^{j\omega})$$

F_k and H_k can be designed such that

$$\frac{1}{M}\sum_{k=0}^{M-1} H_k\left(e^{j(\omega+\frac{2\pi\ell}{M})}\right)F_k(e^{j\omega}) = \begin{cases} G(e^{j\omega}), & \text{if } \ell = 0 \\ 0, & \text{otherwise} \end{cases}$$

Therefore: $Y(e^{j\omega}) = G(e^{j\omega})\,U(e^{j\omega})$

$$F_k(e^{j\omega}) = e^{-j\omega(M-1-k)}$$
$$H_k(e^{j\omega}) = e^{-j\omega k}$$
⇨ $G(e^{j\omega}) = e^{-j\omega(M-1)}$ ⇨ $y(nT) = u((n-M+1)T)$

Hybrid Filter Banks

Effects of gain mismatches: $x_k(nT) = (1+a_k)v_k(nT)$

Output spectrum becomes:

$$Y(e^{j\omega}) = \frac{1}{M}\sum_{\ell=0}^{M-1} U\left(e^{j(\omega+\frac{2\pi\ell}{M})}\right)\sum_{k=0}^{M-1}(1+a_k)H_k\left(e^{j(\omega+\frac{2\pi\ell}{M})}\right)F_k(e^{j\omega})$$

or

$$Y(e^{j\omega}) = \left(G(e^{j\omega}) + \Lambda_0(e^{j\omega})\right)U(e^{j\omega}) + L(e^{j\omega})$$

where

$$L(e^{j\omega}) = \frac{1}{M}\sum_{\ell=1}^{M-1} U\left(e^{j(\omega+\frac{2\pi\ell}{M})}\right)\Lambda_\ell(e^{j\omega})$$

☜ Aliased components

$$\Lambda_\ell(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1} a_k H_k\left(e^{j(\omega+\frac{2\pi\ell}{M})}\right)F_k(e^{j\omega})$$

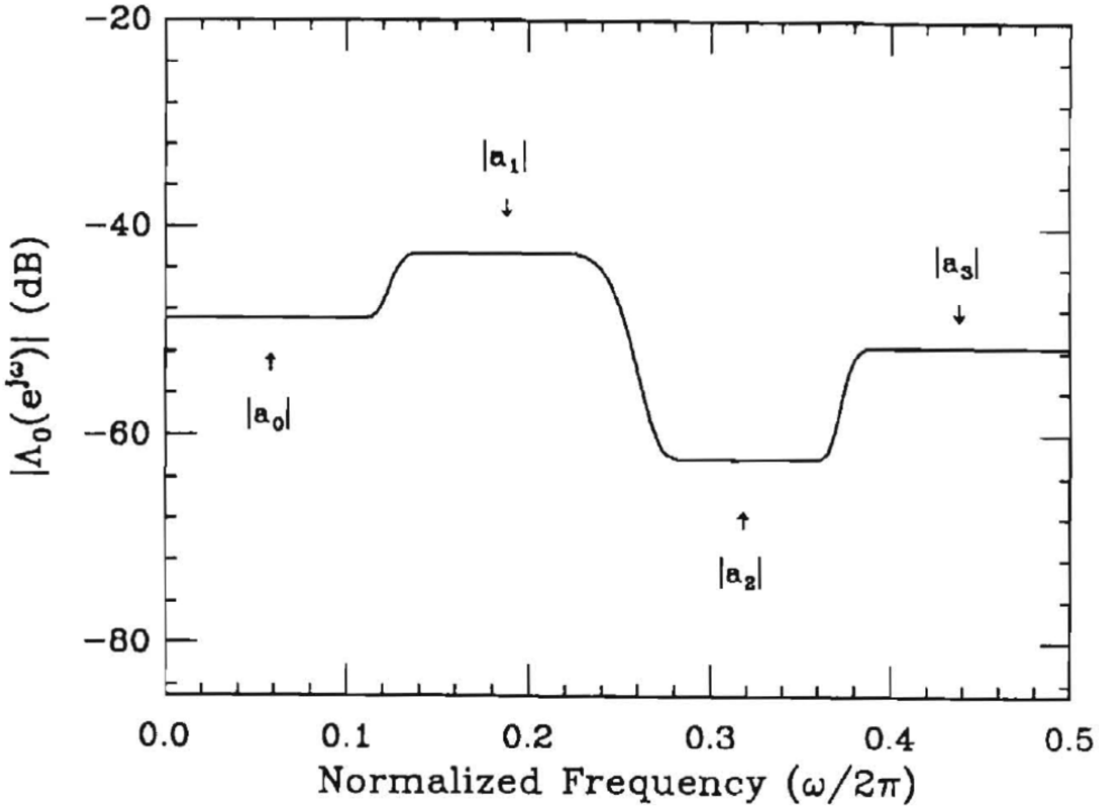
Hybrid Filter Banks

Effects of the Gain Term $\Lambda_0(e^{j\omega})$

$$\left|\Lambda_0(e^{j\omega})\right| \approx |a_k|,\ \omega_k < \omega < \omega_{k+1},\ k = 0,1,\ldots,M-1$$

$$\Rightarrow\ \left|\Lambda_0(e^{j\omega})\right| \ll \left|G(e^{j\omega})\right|,\ 0 < \omega < \pi$$

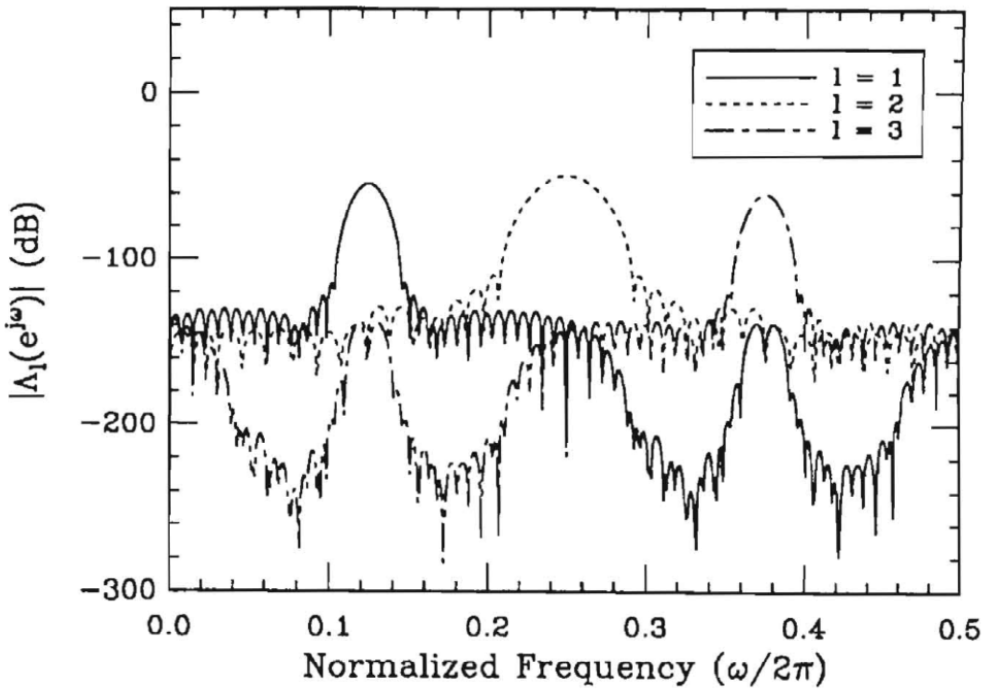
Ex.: $\sigma_a = 0.01$



Hybrid Filter Banks

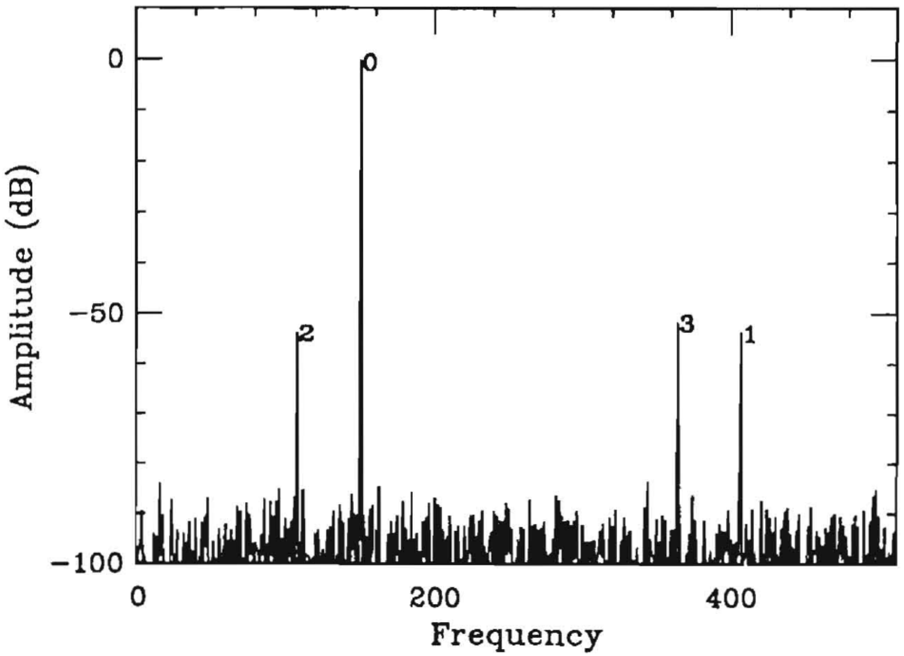
Effects of the Aliasing Terms $\Lambda_l(e^{j\omega})$

Ex.: $\sigma_a = 0.01$



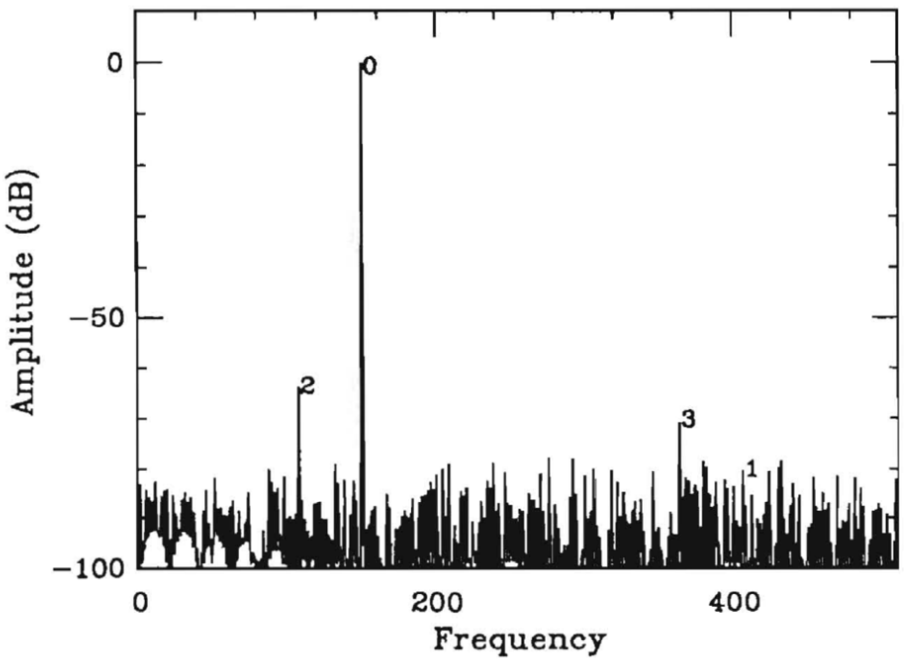
Comparisons

Time interleaved A/D; M = 4; σ_a = 0.005.
1024-point FFTs; ω_o = 149/1024.



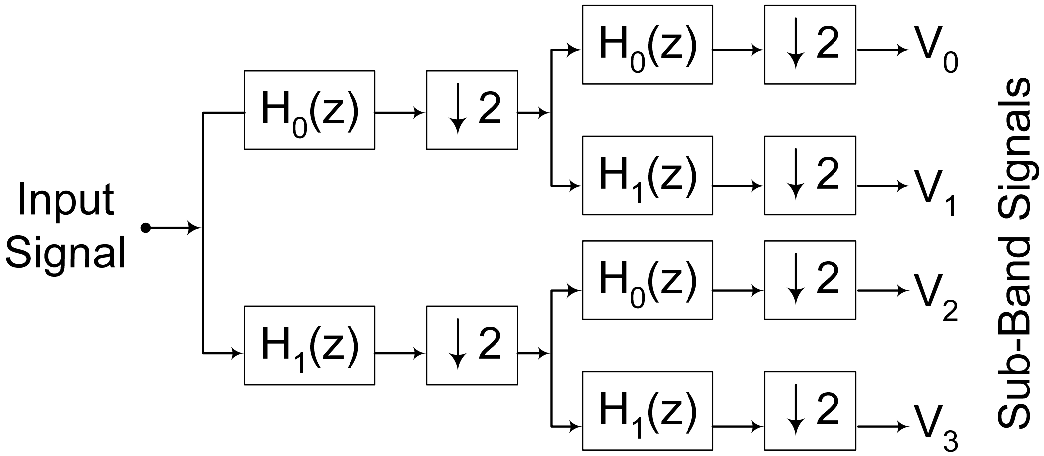
Comparisons

HFB-Based A/D; M = 4; σ_a = 0.005.
1024-point FFTs; ω_o = 149/1024.



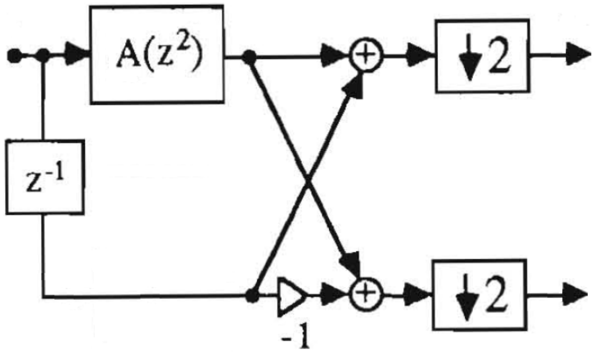
Analysis Filter bank Design

Tree-structured analysis filter bank; M = 4.



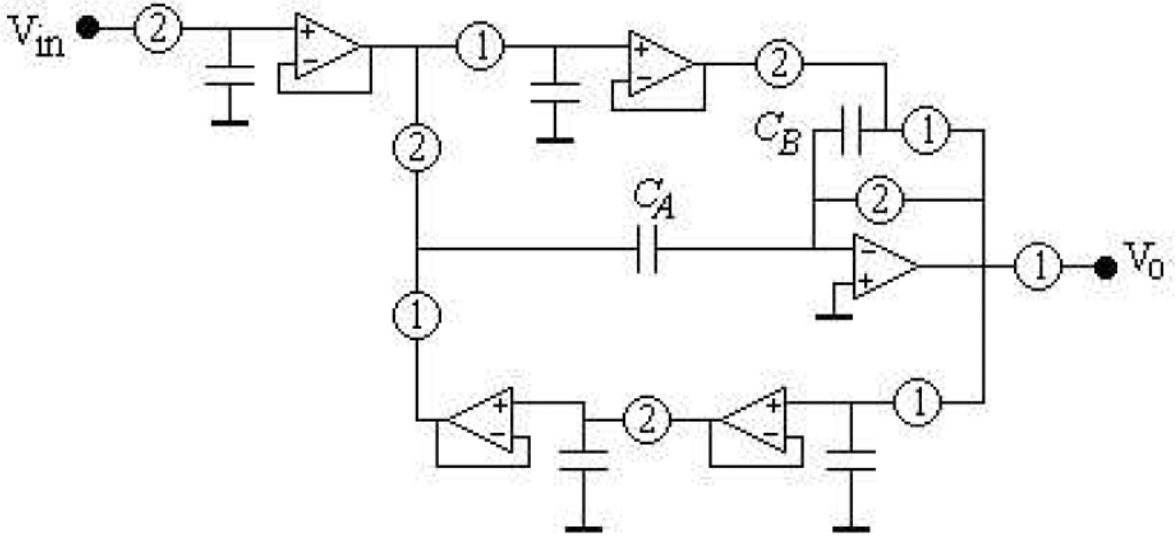
Analysis Filter Bank Design

Switched-capacitor realization



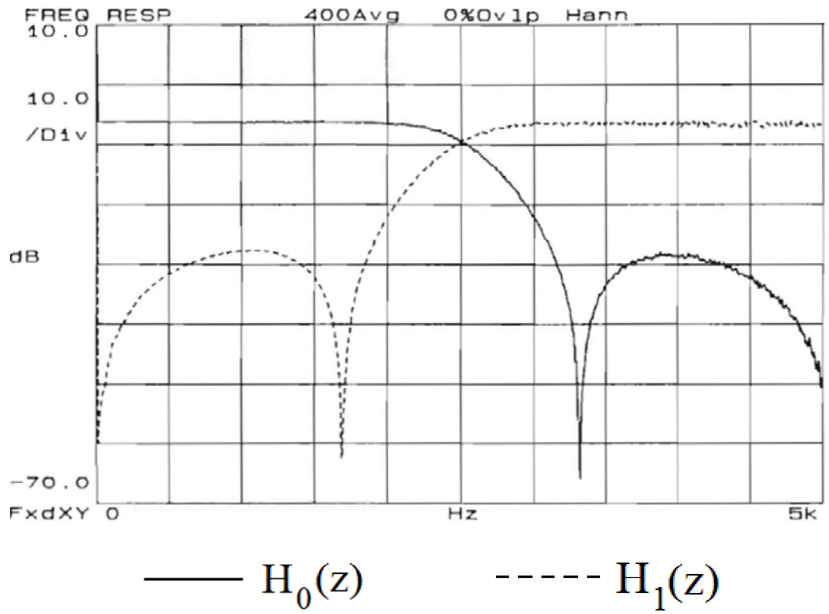
$$A(z) = z^{-1/2} \frac{z^{-1} + C_A/C_B}{1 + (C_A/C_B)z^{-1}}$$

Allpass section



Experimental Results

Measured Frequency Responses



$C_A / C_B = 2$

Comparisons

HFB-Based A/D; M = 4; σ_a = 0.005.
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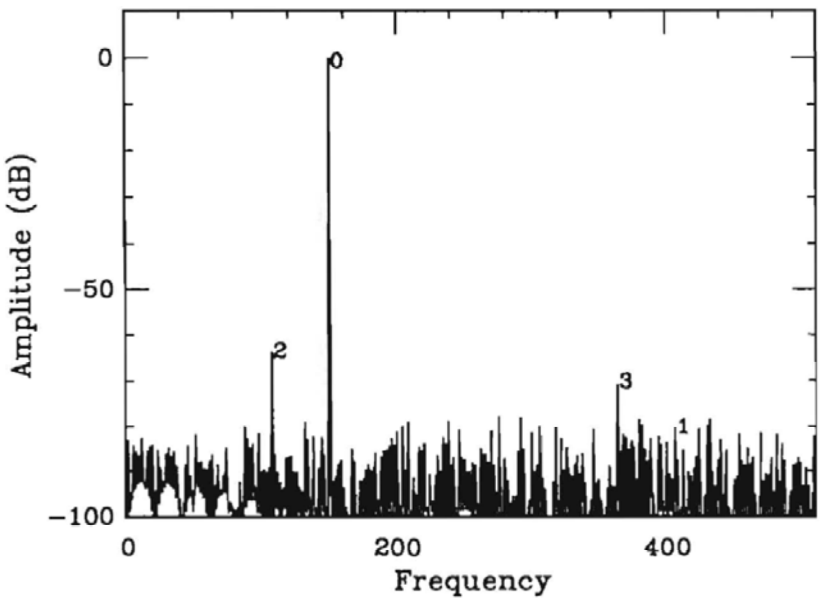
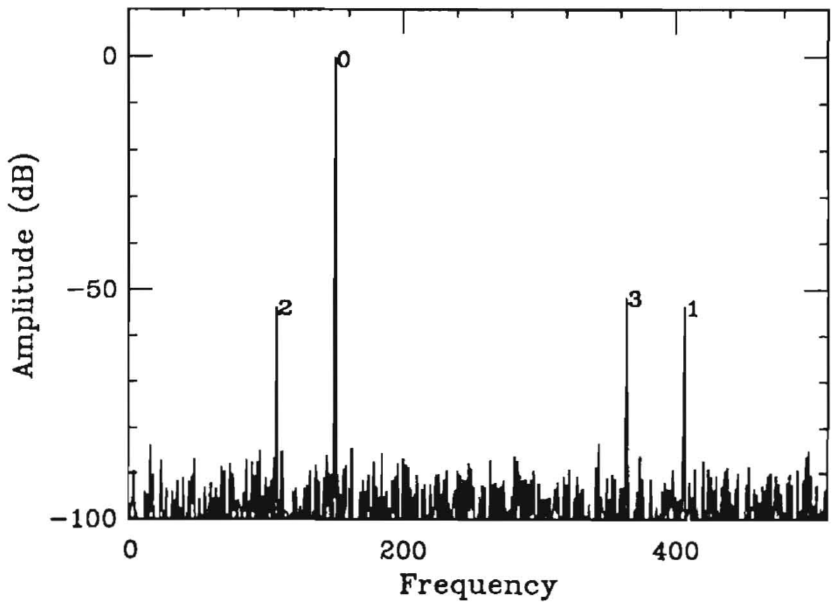
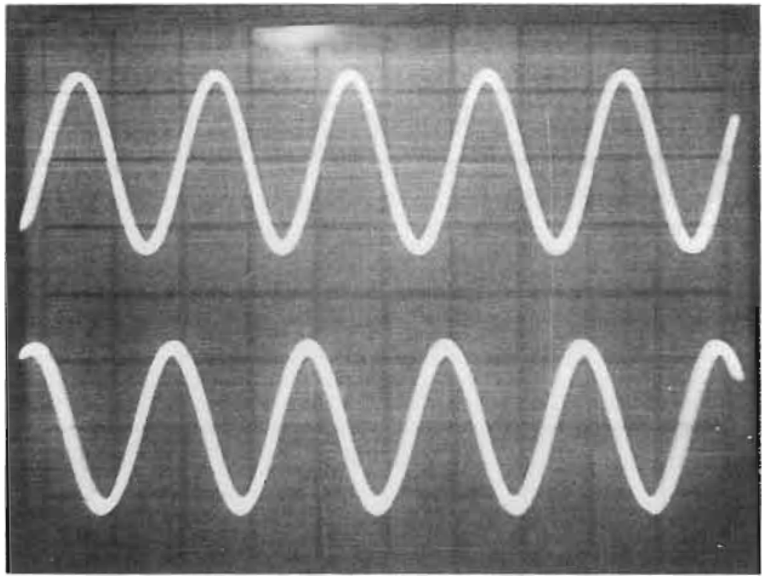


Time interleaved A/D



Experimental Results

Experimental Results



Vertical scale: 2V/div
Horizontal scale: 0.2 ms/div

Conclusions

- **High-speed and high-resolution A/D conversion can be achieved with time-interleaving technique;**
- **Inherent to this approach, mismatches among the signal paths constitute a serious drawback;**
- **A large number of methods have been proposed to reduce their effects, mostly based on extensive compensation in the digital domain of estimated parameters (gain, offset, jitter).**
- **An alternative is the use of hybrid filter banks, which has the advantage of quantization noise control inside each frequency band;**
- **The cost is increase in circuit complexity, and its consequences (noise, distortion, power, etc …)**